

Investigation On Cornu's Spiral And Fibonacci Numbers In The Evolution Of Molluscan Shells.

Konwar R, Baruah. G. D.

Department of physics, Tinsukia College, Tinsukia-786125, India.
Centre for Laser and Optical Science, New Uchamati; Doom Dooma-786151, India
E-mail: konwar_rajib@rediffmail.com

ABSTRACT: The present work describes the growth of a particular species(Uno) of molluscan shell in terms of moving vectors which ultimately give rise to phasor diagram similar to a Cornu's spiral in optics. The appearance of the so called Fibonacci numbers in molluscan shells has been identified.

Keywords : Molluscan shells, Fibonacci numbers.

1. Introduction:

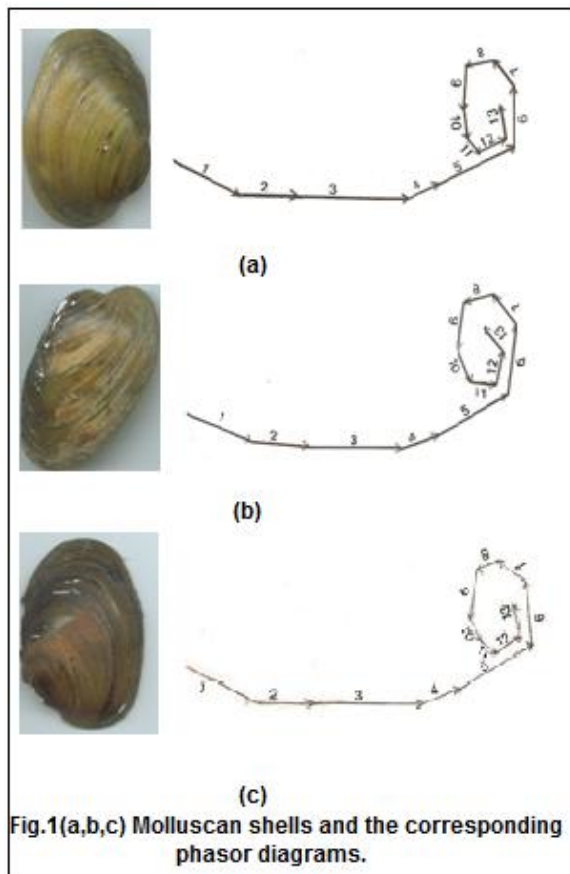
Sea shells and molluscan shells have attracted the attention of researchers for many interesting physical phenomena exhibited by these naturally occurring objects. The materials of the molluscan shells present different appearances at different cases. It is hard, and white in shark, translucent in window pane oyster and beautifully lustrous and iridescent in mother of pearl. Little is known of the process of calcium carbonate formation and crystallization in the molluscan shells. Nature has numerous examples that suggest that biological synthesis might be in many ways superior to conventional synthesis of materials by human beings. The shell of a molluscan built out of calcium carbonate has 3000 times higher fracture resistance than crystals of calcium carbonate. Shell growth involves increase in both area and thickness. Area increase is a function of increase of mantle and increase in thickness relates to the deposition of calcium carbonate and organic matrix. The reference points of all analyses of Bivalve shell growth are the "umbones"; the oldest part of the shell, situated more or less anteriorly above the hinge line. It is to be noted that many shells display concentric rings surrounding the "umbones" and they are known as growth lines. Most shells are not symmetrical about a line vertically downward from the "umbone". The majority of bivalves contain chromo proteins associated with the conchiolin of the bivalves and many of the melaninoid pigments may owe their origin to the biochemical activities associated with the quinine tanning of conchiolin. Several decades ago Raman and his coworkers¹⁻⁵ investigated experimentally various optical properties like iridescence and geometrical forms associated with shells. According to Raman the differences in optical properties are due to the atomic structure of the materials. Raman was particularly interested in the geometrical form of shells and carving of their external surfaces. While one may not know yet whether the beautiful patterns on the shell surfaces have a functional role one is at present time at least able to explain how such patterns could arise. As in the case of Leisegang precipitation^{6,7}, it appears that certain nonlinearities called into play during cell growth are the controlling factors. Meinhart and Klingler⁸ have succeeded in simulating many of the observed shell pattern using a suitable non-linear reaction diffusion model. In the bio-mineralization field, the molluscan shell is one of the best studied of all calcium carbonate bio-minerals. Particular attention has been given to organic matrix⁹⁻¹³. The nacreous layer of Molluscan shell

has been studied extensively for several decades, particularly with X-ray and electron microscope technique. This work has been largely successful in describing microstructure of nacre¹⁴⁻¹⁸. The mechanism of growth of nacreous layer is complex and is not completely understood in spite of investigation of several decades. It was Kitano and Hood¹⁹ who first showed that aragonite is the most favorable phase of calcium carbonate (CaCO_3) to nucleate in sea water supersaturated with respect to that mineral. The submicroscopic structure of the conchiolin of the prisms in Molluscan shells has been extensively studied several decades ago by Gregoire²⁰. It was established that the inorganic part of a prism is made up of calcite crystals piled one upon the other, the pile being wrapped into a conchiolin sheath built upon a fibrillar structure covered by a very dense protein component. The active role of proteins in bio-mineralization is a fundamental issue and it has been described by several workers^{21, 22}. It represents a source of inspiration for nanotechnology. The "brick and mortar" ordering of nacre (mother of pearl,) has already inspired the toughening of ceramic materials by co-processing rigid ceramics as silicon carbide and compliant interlayer as boron nitride²³. The spatial organization of the bio-mineralization mechanism of the crystal growth is the main focus of the recent works²⁴. Recently laser induced fluorescence spectra of few species of molluscan shells were investigated by Konwar. et. Al²⁵. We have indicated here a brief overview of the works and investigation carried out on biomineralization particularly on molluscan shells during last several decades. In the present work we are concerned with the subject of evolution of the growth pattern of the shells belonging to a particular species and their representation in terms of phasor diagrams which are quite analogous to Cornu's Spiral in optics. We also discuss specifically the issue of Fibonacci numbers which appear in certain specimens of molluscan shells. From the survey of available literature it is reasonable to believe that these are the first reports of this kind.

2. Evolution of molluscan shells, phasor diagram:

In the present work we reproduce the photographs of the molluscan shells which have been procured from a place known as "Maguri Bill", near Dibru Saikhowa National Park of Tinsukia District, Assam. For convenience only three specimens are shown along with the phasor diagrams which are so similar to Cornu's Spiral in optics. It is

worthwhile to elaborate here how these phasor diagrams are constructed. The vectors are drawn by taking the centre of an ellipse as a starting point of a vector and connecting the successive centres of the ellipses which are visible on the projections of the photographs of the molluscan shells. It is worthwhile to note that about thirteen vectors can be drawn with arbitrary magnitude as distances between two centres of the ellipses. As the vectors change direction they ultimately evolve as phasor diagram very similar to Cornu's Spirals in optics²⁶. It is worthwhile to note that in Fig.1 we have only shown three representative diagrams corresponding to two specimens of molluscan shells. It is possible to construct such diagrams for other samples as well.



3. Molluscan shells and Fibonacci numbers:

In this section we describe the appearance of a mathematical series and the so called Fibonacci numbers in some molluscan shells. The Fibonacci numbers are the sequence of numbers defined by the linear recurrence equation

$$F_n = F_{n-1} + F_{n-2} \quad \dots \dots \dots (1)$$

with $F_1 = F_2 = 1$. As a result of the definition (1) it is convenient to define $F_0 = 0$. The Fibonacci numbers for $n = 1, 2, 3, \dots$ are $1, 1, 2, 3, 5, 8, 13, 21, 34, \dots$. These numbers seem to appear in the Nature in numerous cases. It is worthwhile to note that the succeeding terms of the system are obtained by the sum of two preceding terms being with the lowest whole numbers. Thus we have the series of numbers $1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$. This is also known as Fibonacci series. This series was first discovered by at 17th century called Fibonacci Gerard. Its relation to the phenomenon of planet growth is brought out by Church²⁷. There are few other characteristics of the Fibonacci series which are interesting to note. As for instance if we take three consecutive numbers the square of the middle term and the difference between the multiplication of the first and third terms is always the same. Mathematically

$$F_{n-1} \times F_{n+1} - F_n^2 = (-1)^n \quad \dots \dots \dots (2)$$

Again, leaving aside few small terms, if we take the ratio of two consecutive terms then the ratio always remain the same, and it is equal to 1.618. This number is known as the divine number. Fig.1 shows the photographs of few specimens of shells. If we take the semi-major axis of the successive prominent elliptical rings we observe that the ratio of the distances (taken arbitrarily in millimeters) of the semi-major axes remain nearly constant and approaches the so called divine number as indicated earlier. Table I shows few measurement.

Table I

Ellipse No.	Semi major axis (in mm)	Ratio of successive rings	Difference from the divine number (1.6180)
1	3.2	0	—
2	5	1.5625	- 0.0555
3	8	1.6000	- 0.018
4	13	1.6258	+ 0.009
5	21	1.6153	- 0.003

4. Summary and conclusion:

In the present work we introduce the concept of moving vectors which give rise to phasor diagrams similar to Cornu's Spirals in optics. It is also reasonable to believe that molluscan shells exhibit Fibonacci numbers in their evolution.

For convenience only three specimens are shown along with the phasor diagrams which are so similar to Cornu's Spiral in optics. It is worthwhile to elaborate here how these phasor diagrams are constructed. The vectors are drawn by taking the centre of an ellipse as a starting point of a vector and connecting the successive centres of the ellipses which are visible on the projections of the photographs of the molluscan shells. It is worthwhile to note that about thirteen vectors can be drawn with arbitrary magnitude as distances between two centres of the ellipses. As the vectors change direction they ultimately evolve as phasor diagram very similar to Cornu's Spirals in optics²⁶. It is worthwhile to note that in Fig.1 we have only shown three representative diagrams corresponding to three specimens of molluscan shells. It is possible to construct such diagrams for other samples as well.

REFERENCES:

1. C V Raman, Proc. Ind. Acad. Sci. **A1**, 859(1934).
2. C V Raman, Proc. Ind. Acad. Sci. **A1**, 567(1934).
3. C V Raman, Proc. Ind. Acad. Sci. **A1**, 574(1934).
4. C V Raman and D Krishnamurti, Proc. Ind. Acad. Sci. **39**,1, (1954).
5. G. Venkataraman, "Journey into light" Life and Science of Sir C V Raman, Penguin Book India (P) Ltd. (1994) p479.
6. K Subba Ramaiah, Proc. Indian Acad. Sci. **A9**, 455(1939).
7. K Subba Ramaiah and C V Raman, Nature (London) **142**, 355(1938).
8. H. Meinhard and M Klingler, J. Theo. Bio **126**, 63(1987).
9. M A Crenshaw, Biomineralization, **6**, 6-11(1972).
10. G Kranmpitz, J. Engels and C Cazawx, "Biochemical studies on water soluble proteins and related components of gastropod shells" In the mechanisms of Mineralization in the Invertebrates and Plants. (Watabe, N & Wilbur, K M, eds), pp 175-190. The B. W. Barush Libray in Marine Science no. 5, The University of South Carolina Press, Columbia.
11. S. Weiner, W. Traub and H A Lowenstam. Organic Matrix in calcified exoskeletons. In Biomineralization and Biological Metal accumulation. (Westbrock, P& DeJong, E.W eds) pp-205-224(1983), Reidel, Wordrecht, Holland.
12. J Keith, D Stockwell, D Ball, K Remillard, D Kaplan, T Thannhauser and R Sherwood, Comp. Biochem. Physiol. **105B**, 487(1993).
13. Y Levi-Kalisman, G Falini, L Addali & S Weiner J. Struc. Biol. **135**, 8-7(2001).
14. H A Lowenstam and S Weiner, On Biomineralization, Oxford University Press, New York, (1989).
15. S Weiner CRC Crit Rev. Biochem, **20**, 356-408(1986).
16. N Watabe. Prog. Crystal Growth Charact, **4**, 99-147(1981).
17. K M Towe, Bull. Natl. Pearl Res. Lab. JPN **7**, 703(1961).
18. S. W. Wise Eclogae Geol. Helv. **63**, 775-797(1970).
19. Y Kitano and D W Hood; J. Oceanogr. Soc.. JPN **18**, 35-39(1962).
20. C. H. Gregoire, J. biophys. Biochem. Cytol. **9**, 395-400(1960).
21. G Falini, S. Albeck, S. Weiner and L. Addadi; Science **271**, 67-69 (1996).
22. R. Naslain, R. Paillar, X. Bourrat, and F. Heurtevent, Mimicking the layered structure of natural shells as a design approach to fiber matrix interface in Proceed. ECCM-8 (J. Girelli – Visconti ed.), Vol.4, Woodhead Publ CMCs,, Abington, Cambridge, UK pp 191-199 (1998).
23. B L Smith, Chemistry and Industry **16**, 649-653(1998).
24. Marthe Rousseau, Evelyne Lopez, Alen Coute, Gerard Mascarel, David C Smith and Xarier Boarrat, 9th International Symposium on Biomineralization, 6-9 December(2005) Pucon, Chile.
25. R.Konwar, R Changmai and G D Baruah; Indian J. Pure & Appl. Phys. **42**, 812-815 (2004).
26. F. A. Jenkins and H. E. white, Fundamentals of Optics; McGRAW – HILL International Editors (1981) p389.
27. A H Church, On the Relation of Phyllotaxis to Mechanical Law, (Williams and Norgate, London) 1904.