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On Application Of Matlab To The Motion Of A Finitely Damped String On Sobolev Spaces

Egbuhuzor Udechukwu P., **Ugbene Ifeanyichukwu**, **EzeEverestus O**

Department of Mathematics, Caritas University, Amorji – Nike, Enugu State, Nigeria
Department of Mathematics, Micheal Okpara University of Agriculture, Umudike, UmuahiaAbia State, Nigeria.
E-mail: udegod2002@yahoo.com

ABSTRACT: The problem of vibrations of plucked strings was investigated. The objective is to determine the vibrational behaviour of a guitar with nylon and steel strings. The derived Newton's second law of motion [7] with damping coefficient was used and the equation of the form

$$-T(x)\sin\theta + T(x+\Delta x)\sin\beta - \omega\Delta x U_t = mU_{tt} \quad . . . \ (1)$$

Was considered. The method of separable variable was applied and we obtained

$$U_{xx} - KU_t = \frac{1}{C^2}U_{tt} \quad . . . \ (2)$$

The result shows that nylon strings decay mainly as a result of internal damping in the strings while steel strings decay due to air viscosity. Therefore we conclude that nylon strings are under less tension than steel strings [5].

Keywords and phrases: motion, Sobolev space, mechanical system, vibrations, damped strings, plucking technique, nylon and steel strings. 2010 mathematical subject classification: 70E15, 70E18, 70F25

INTRODUCTION

In this paper, we considered mechanical system because its dynamic behaviour is resolved by interaction of several components. The plucked strings only radiate a small amount of sound directly, but they excite the bridge and the top plate, which in turn transfers energy to the air cavity, ribs and black plates [2]. Sound is radiated efficiently by the vibrating plates and through the sound hole [3]. There are three major techniques through which string instruments produce sound from one or more vibrating strings, transferred to the air by the body instrument, plucking, bowing and striking [2] Here, we are concerned with plucking technique since guitar undergoes that.

METHODOLOGY

In the derivation of model of a damped guitar string, we assumed there is an additional force in the vertical direction that is proportional to the infinitesimal piece of string's velocity with width Δx [7]. The derived Newton's second law of motion with the damping coefficient as in equation (1) would be applied. This led us to a system of second order partial differential equation when solved. The analytical method of separable variable of solving a problem of one-dimensional finitely damped string (2) was used. We also used laboratory experiment values generated by [5] to run a graphical solution of (8) using MATLAB programming to determine the vibrational behaviour of nylon and steel strings. Our interest is not in the musical effect but the basic mechanical fact that a string with both ends fixed has a number of well-defined states of natural vibrations. This produces a simple wave which must be subject to a restoring force that continually pulls the system forward to equilibrium position. Virtually every system possesses the capability of vibration, and most systems can vibrate freely in a large variety of ways [1], when the basic law of nature are applied to dynamic systems with restoring forces, we usually get the partial differential equations of the form.

$$U_{xx} - KU_t = \frac{1}{C^2}U_{tt} . . . \ (2)$$

The solutions of the differential equation are used to develop theories which can be used to analyze general solution [2]. [3] said that vibration deals with oscillatory motion of dynamic systems and forces associated with them. [4] in the model of vibrating string, investigated the effect of pluck point and string, investigated the effect of pluck point and string properties on the sound of the string. He also looked at how the sound of the guitar string evolves over time from the pluck to when the sound has finally died out.

MAIN RESULT

Evoking our equation (1),

$$-T(x)\sin\theta + T(x+\Delta x)\sin\beta - \omega\Delta x U_t = mU_{tt}$$

Where ω is the constant of proportionality, $\tan\theta$and tanβ are the slopes of the tangent lines. Solving the equation, we obtained (2) Where k is the damping coefficient. Solving (2) using separable variable method of a boundary value problem,

Let $U(x,t) = X(x)T(t)$

$U_x = X'T;\ U_{xx} = X''T; U_t = XT';\ U_{tt} = XT''$

Using the initial conditions,

$$U(x,0) = \begin{cases} \frac{h}{b}x; & 0 \le x \le b \\ \frac{h}{b-L}(X-L); & b \le x \le L \end{cases} \dots \ (3)$$

Applying (3) on (2), we obtain a general solution which is our main result:

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$U(x,0) = \sum_{n=1}^{\infty} e^{-\alpha_n t} \left(a_n \cos \mu_n t + b_n \sin \mu_n t\right). \sin \frac{n\pi x}{L} \ldots(4) = \sum_{n=1}^{\infty} \frac{e^{-\alpha_n t}}{b(b-L)n^2\pi^2} \left\{2 \cos \mu_n t + \frac{KC^2}{\mu_n} \sin \mu_n t\right\}. \sin \frac{n\pi b}{L}. \sin \frac{n\pi x}{L}$ ... (5)

Where $b_n = \frac{KC^2 a_n}{2\mu_n}$ and $a_n = \frac{-2hL^2}{b(b-L)(n\pi)^2} \sin \frac{n\pi b}{L}$.

DISCUSSION

The derived Newton's second law of motion which resulted to second order partial differential equation was solved. MATLAB Programming was used to run a graph on a general solution of damped one-dimensional wave equation for nylon and steel strings at each pluck points $(x = 0.12, 0.225, 0.35, 0.485, 0.6, 0.725, \text{and } 0.83)$ but our main focus is on the pluck point at the middle$(x = 0.485)$. This authenticates the statement made by [5] that in a nylon string guitar, the higher modes decay mainly as a result of infernal damping in the string, but a steel string guitar decay as a result of air viscosity. This is to say that the steel strings are actually damped less by air viscosity than nylon strings since they are thinner.

CONCLUSION

The importance of decay rate is to make sure that the sound produced by the guitar dies out. The way forward to this paper is to determine how the decay rate can be captured mathematically since [4] could only list the values which helped us in running the graphical solution without showing any mathematical or laboratory derivations. It is therefore true that nylon strings are under less tension than steel strings. The greater tension would be requiring more pressure to depress the strings.

MATLAB CODES FOR ONE-DIMENSIONAL FINITELY DAMPED STRING (NYLON & STEEL)

```
Clear

clc

Alpsteel = [0.1 0.13 0.16 0.18 0.21 0.22 0.23 0.25 0.27 0.28];

Alnylon = [0.4 0.55 0.7 0.9 1.05 1.15 1.3 1.45 1.55 1.65];

n=1:10;

K=0.026;

C=11.7;

X= [0.12 0.225 0.35 0.485 0.60 0.725 0.83];

L=0.965;

h=1;

b=0.4825;

Yn=n*pi/L

Un=sqrt ((( 4*Yn.^2*C^2) – (K^2*C^4))/4)

An= (2*h*L^2*sin(n*pi*b))./(b.*(L−b).*((n.*pi).^2).*L)

Bn= (K*C^2./2*Un).*An

i=1

for j= 1:7

for t= 0.01:0.01:1

Unylon (i)= sum (exp(−Alpnylon.*t).*((An.*cos(Un.*t)) + (Bn.*sin (Un.*t))).*sin (n*pi*X (j) /L))

Usteel (i)= sum (exp(−Alpsteel.*t).*((An.*cos(Un.*t)) + (Bn.*sin (Un.*t))).*sin (n*pi*X (j) /L))

i= i+1

end

t= 0.01:0.01:1

figure

plot (t, Unylon)

title (sprintf ( ′Nylon \n Displacement at x=%12.8f ′,

x (j)))

xlable ( ′Time (secs) ′)

figure

plot (t, Usteel)

title (sprintf ( ′Steel \n Displacement at x=%12.8f ′,

x (j)))

xlable ( ′Time (secs) ′)

i=1

end
```

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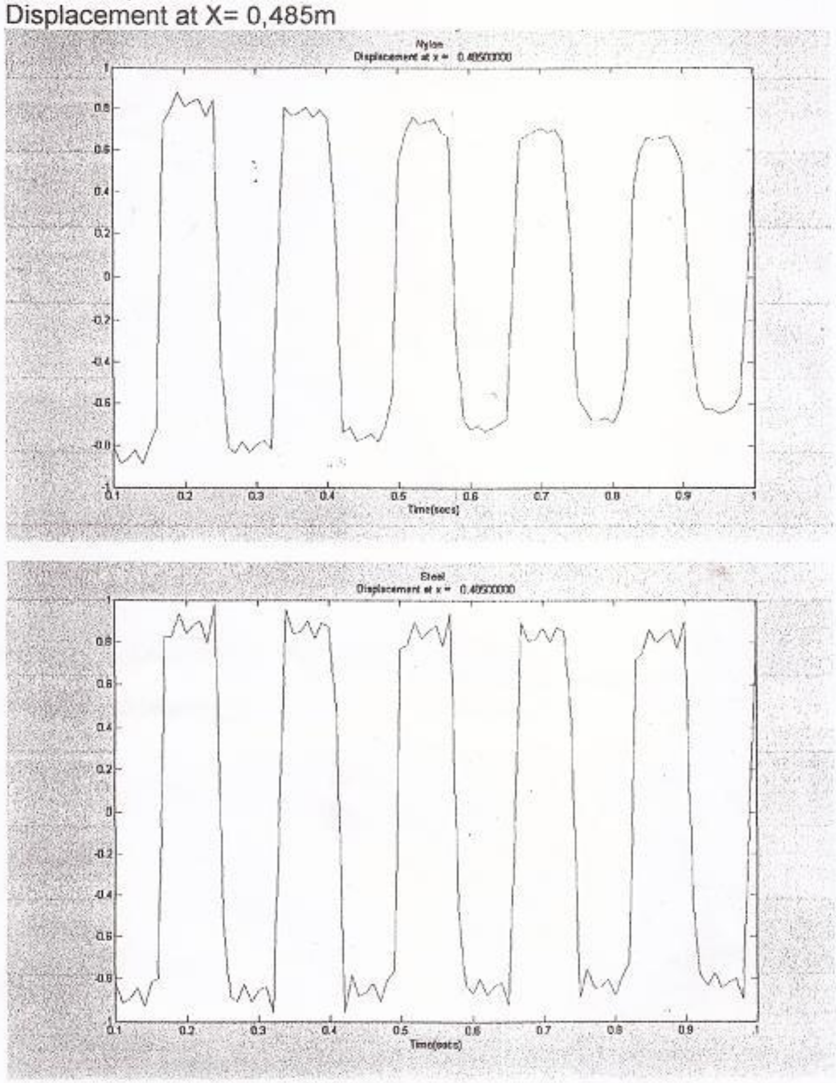
Displacement at X= 0,485m



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