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On Lucas Sequence Formula For Solving The Missing Terms Of A Recurrence Sequence

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ABSTRACT: Lucas sequence is associated with Fibonacci sequence. Many identities have been already derived for Lucas Sequence in books and mathematical journals. This paper will present formulas in solving Lucas means as well as solving the sequence itself. 2010 Mathematics Subject Classifications: 11B50, 11B99

Key Words and Phrases: Lucas Sequence, Lucas like sequence, missing terms.

1. INTRODUCTION

Natividad [1]presents a formula for solving the missing terms of a Fibonacci-Like sequence given a first term a and a last term b.Also ,he provides a solution for solving Lucas sequence. He shows that the formula for the first term, denoted by x_1,for solving the missing terms of a Fibonacci-Like sequence with term a and last term b is given by

$$x_1 = \frac{b - F_n a}{F_{n+1}}$$

where n is the number of missing terms and $F_n$is the n th term of the Fibonacci sequence.

On the other hand,the formula for solving Lucas can be found using the formula for $x_1$defined by

$$x_1 = \frac{b - L_n a}{L_{n+1}}$$

Where L_n is the n-th term of the Lucas sequence.

Natividad have derived the formula for the second term,$W_1=x_1$ of the sequence dependent on the first and last term by looking at each case of the number of missing terms. He also provided a table for the coefficient of a in the numerator and the coefficient of the denominator and looked for a pattern as the number of missing terms increases. In this note, we will derive the formula for solving the missing terms of the sequence by using the Binet`s formula of the recurrence relation and using the fact that b is the $(n+1)^{th}$ term of the sequence with a as the first term. Thus, here we present the same results shown by Natividad but of different approach

2 PRELIMINARIES

Definition 2.1 The Lucas numbers are a sequence which is formally defined recursively as:

$L_0=2$; $L1=1$; $L_n = L_{n-1}+L_{n-2}$.

Compare the Fibonacci numbers. We can see that the first few Lucas numbers are:

2,1,3,4,7,11,18,29,47,76,123,199,322,521,843,…

Before we proceed for the formula on solving the missing terms of a Fibonacci-like sequence as well as the formula for solving the Lucas sequence ,we consider the second order linear recurrence sequence $\{W_n\}$ defined by the recurrence relation $W_{n+1} = pW_n + qW_{n-1}$,with $W_0=a$,$W_1=c$,where a, c and p, q are arbitrary real number for n>0 The Binet`s formula for the recurrence *sequence* $\{W_n\}$ is given by

$$W_n = \frac{A\alpha^n - B\beta^n}{\alpha - \beta} \quad \ldots(1)$$

Where $A = c - a\beta$, $B = c - a\alpha$

Take note that the generating function of $\{W_n\}$ is a quadratic equation of the form, $x^2 = px + q$ .Solving for its roots we obtain,

$$\alpha = \frac{p + \sqrt{p^2 + 4q}}{2} \text{ And } \beta = \frac{p - \sqrt{p^2 + 4q}}{2}.$$

Notice that $\alpha + \beta = p,\ \alpha - \beta = \sqrt{p^2 + 4q},$ and $\alpha\beta = -q,$

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The whole purpose of considering the second order linear recurrence sequence $\{W_n\}$ is to use its two special cases. When p=1 and q=1 , W_n is the n-th term of the Fibonacci – like sequence when p=2 and q=1, W_n becomes the n-th term of the Lucas sequence .In terms of recurrence relation, we have the following special cases: (1) if p=1 and q=1 we will have a Fibonacci like sequence defied by

$W_{n+1} = F_{n+1} = F_n + F_{n-1}$ Where W_0 = a, W_1 = c …(2)

(2)if p =2 and q = 1 ,We will obtain a Lucas sequence given by

$L_n = L_{n-1} + L_{n-2}$. Where W_0 = a, W_1 = c ….(3)

In the following section we shall provide an explicit formula for solving the general term W_n of the recurrence relation (2) and (3) using equation (1) as part of the proof of the following formula

$x_1 = \frac{b - F_n}{F_{n+1}}$ for Fibonacci –like sequences and

$x_1 = \frac{b - L_n}{L_{n+1}}$ for Lucas sequence

3. MAIN RESULTS

We will use the Binet`s formula for the second order linear recurrence sequence $\{W_n\}$ of the two special cases mentioned above to prove the following theorems.

Theorem3.1.For any real number a and b

$$x_1 = \frac{b - F_n a}{F_{n+1}}$$

….(4)

Where n is the number of missing terms F_n is the n-th Fibonacci number and a and b is defined as the first term and the last term of the sequence respectively.

Proof: Let W_0 = a be the first term of the sequence and b be the last term of the sequence. If n is the number of missing terms between a and b then, $b=W_{n+1}$.Now suppose that x_1 is the second term in the sequence, hence the Binet`s formula for the sequence is given by

$$W_n = \frac{(x_1 - a\beta)\alpha^n - (x_1 - a\alpha)\beta^n}{\sqrt{5}}$$

Where α is equal to ϕ known as the golden ratio and $\beta = 1 - \phi$ This would imply that

$$b = \frac{(x_1 - a\beta)\alpha^{n+1} - (x_1 - a\alpha)\beta^{n+1}}{\sqrt{5}}$$

$$b = \frac{(\alpha^{n+1} - \beta^{n+1})x_1 - (\alpha^{n+1}\beta - \alpha\beta^{n+1})a}{\sqrt{5}}$$

$$b = \frac{(\alpha^{n+1} - \beta^{n+1})x_1 - (\alpha^n - \beta^n)a\alpha\beta}{\sqrt{5}}$$

$$= F_{n+1}x_1 + F_n a$$

This proves equation (4).

Theorem 3.2. For any real number a and b

$$x_1 = \frac{b - L_n a}{L_{n+1}}$$

Where n is the number of missing terms L_n is the n-th term of the Lucas number and a and b is defined as the first term and the last term of the sequence respectively.

Proof:-.The proof is similar to the previous theorem. Using the Binet`s formula for the Lucas sequence with first term W_0=a and last term b,we have

$$W_n = \frac{(x_1 - a\alpha)\alpha^n - (x_1 - a\alpha)\beta^n}{2}$$

Where α is equal to $1+\sqrt{2}$ known as the silver ratio and $\beta = 1 - \alpha$

Letting n be the number of missing terms between a and b,we obtain

$$b = W_{n+1} = \frac{(x_1 - a\beta)\alpha^{n+1} - (x_1 - a\alpha)\beta^{n+1}}{2}$$

$$b = \frac{(\alpha^{n+1} - \beta^{n+1})x_1 - (\alpha^{n+1}\beta - \alpha\beta^{n+1})a}{2}$$

$$b = \frac{(\alpha^{n+1} - \beta^{n+1})x_1 - (\alpha^n - \beta^n)a\alpha\beta}{2}$$

$$= L_{n+1}x_1 + L_n a$$

Equation (5) follows.

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