The Use Of Numerical Techniques To Analyze The Transient Response Of A Reservoir Undergoing Meor Process

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ABSTRACT: The development of a three-dimensional, three-phase, multicomponent numerical model to analyse the microbial transport and fluid flow phenomena in a microbe-flooded reservoir is presented. The multiphase flow equations are solved in multi cases and in three-dimensional frameworks following the implicit pressure explicit saturation (IMPES) procedure in which the solution of the bacteria transport equations is solved implicitly whereas the nutrient transport equation is solved explicitly. The MEOR model was incorporated into an existing chemical flooding simulator (UTCHEM). The simulation results showed that the sharp increase of pressure after the microbial flooding is caused by pore plugging and the production of biogenic gas. As the time increased and water flooding continued, the permeability recovered slightly with the declogging of microbes in the pore space, resulting in a decrease in pressure drop.

Keywords: Transient Response; Nutrient transport; Bacteria transport; Multiphase flow; MEOR Simulation

1 INTRODUCTION
Microbial Enhanced Oil Recovery (MEOR) is a family of processes that involves the use of selected microbes to achieve enhancement of oil production. The microbial system is carefully designed to produce bioproducts capable of making positive alterations to the inherent rock and fluid properties. The usefulness of this technology lies in the broad range of the bioproducts obtainable and the consequent flexibility it offers the technology. Also advantageous are the economic attractiveness and the environmental friendliness of the process. The first suggestion for using MEOR was made as early as in 1926. The first detailed study of MEOR was conducted by Zobell [1],[2],[3]. He observed that sulfate-reducing bacteria led to gradual separation of oil or tar from sand. Study in this area has been extended by Beck [2] and Updegraff and Wren [3]. Modeling of these processes often rely on conservation laws, incorporating growth and retention kinetics of biomass. A simplified model of this nature was presented by Knapp et al [4]. They predicted porosity reduction as a function of distance and time. Updegraff [3] used a filtration model in order to express bacteria transport as a function of pore entrance size. Similar models were used by Jang et al [5]. In a recent work, Jenneman et al [6] modified the filtration theory to relate permeability with the rate of bacteria penetration. Some of these models were found to show good agreement with experimental results. However none of these models incorporates fundamental laws of bacterial deposition, entrainment or adsorption of bacteria to the rock surface [7]. An early paper that covered all the core phenomena of MEOR and in 3-dimensional formulations was presented by Islam in 1990 [8]. It was based on a conventional three phases black oil formulation and additional equations to describe these processes [8]. The mathematical representation includes equation of species transport, microbial growth kinetics, empirical formula for permeability reduction and continuity equations for pressure and saturation. He also discussed the problem of scaling up of laboratory experiments conducted on MEOR processes.

Another early work by Zhang et al [9] is a one-dimensional model, therefore not suited for field application but for core simulations, nevertheless it has a more advanced formulation of pore-throat plugging and pore surface retention plus a second growth limiting substrate. However, having a reliable simulation model is very crucial in today’s reservoir engineering for field development, scenario runs and sensitivity studies etc. For MEOR, it is more important because it is a complex and interdisciplinary topic that cannot be covered with analytical formulas, rules of thumb, charts or whatsoever [10]. Maudgalya et al [10] in their recommendation have identified an accurate simulation model as key element for the broad success and acceptance of MEOR. For recent, large MEOR pilot projects [11],[12], the application of simulation technology was limited to streamline simulation. The main task is to identify the flow path between the injectors and producers, combined with tracer test to provide the basis for the selection of injector/producer pattern that is used for the project. It is further a basis for verification and analysis of process, later on which can be done for every MEOR project.

2 METHODOLOGY
The multiphase flow equations will be solved in multi cases and in three-dimensional frameworks following the implicit pressure explicit saturation (IMPES) procedure. Also, the solution of the bacteria transport equations will be solved implicitly whereas the nutrient transport equation will be solved explicitly. To achieve this, the equations are discretized in space. This sequence is repeated until convergence is achieved for each time step.

2.1 Model Development
2.1.1 Assumptions
The assumptions made include:

a) No mass transfer between oil and water
b) The gas produced as a metabolic by-product can exist either as a free gas or dissolved gas
c) Isothermal reservoir conditions
d) Uniform gridblocks
e) The fluids are in thermodynamic equilibrium in the reservoir
f) Multiphase flow, that is, simultaneous oil, water and gas flow.

2.1.2 Nutrient Transport Equation
The nutrient transport equation in three-dimension is given by:

\[ \frac{\partial}{\partial t} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial x} \right] \Delta x + \frac{\partial}{\partial y} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial y} \right] \Delta y + \frac{\partial}{\partial z} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial z} \right] \Delta z = \frac{\partial}{\partial t} \left[ \frac{\rho_w}{\rho_w} (1 - \Phi) \frac{\partial C_w}{\partial t} \right]
\]

Applying finite difference approximation to the LHS second order partial differential equation in the x, y and z directions at gridpoints i, j, k respectively [13],

\[ \frac{\partial}{\partial x} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial x} \right] \Delta x_{i,j,k} \approx \Delta_x \left[ \frac{A_w C_w K_w}{\mu_w B_w} \Delta_x \Phi_w \right] \Delta x_{i,j,k}
\]

Also,

\[ \frac{\partial}{\partial y} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial y} \right] \Delta x_{i,j,k} = \left( \frac{A_w C_w K_w}{\mu_w B_w} \Delta_x \Phi_w \right)_{i,j+1/2,k} \Phi_{wx,i,j,k} - \Phi_{wx,i,j,k-1}
\]

Let

\[ T_{wx,i+1/2,j,k} = \left( \frac{A_w C_w K_w}{\mu_w B_w} \Delta_x \Phi_w \right)_{i+1/2,j,k}
\]

Hence, (3) becomes

\[ \frac{\partial}{\partial x} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial x} \right] \Delta x_{i,j,k} = \left( T_{wx,i+1/2,j,k} \Phi_{wx,i+1,j,k} - \Phi_{wx,i,j,k-1} \right)
\]

From (2), (6) can be written as:

\[ \Delta_x \left[ \frac{A_w C_w K_w}{\mu_w B_w} \Delta_x \Phi_w \right] \Delta x_{i,j,k} = \left( T_{wx,i+1/2,j,k} \Phi_{wx,i+1,j,k} - \Phi_{wx,i,j,k-1} \right)
\]

The finite difference approximation to the second order differential term in y-direction, is derived with similar procedure to the just presented approximation for the x-direction term, but in this case, we consider gridpoint (i,j,k) with its gridblock-boundaries (i-1/2,j,k) and (i+1/2,j,k) and its neighboring gridblocks (i-1,k) and (i+1,k) and (i,j-1,k) in the y-direction.

The approximation in the y-direction may be expressed as:

\[ \frac{\partial}{\partial y} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial y} \right] \Delta y_{i,j,k} \approx \Delta_y \left[ \frac{A_w C_w K_w}{\mu_w B_w} \Delta_y \Phi_w \right] \Delta y_{i,j,k}
\]

\[ \Delta_y \left[ \frac{A_w C_w K_w}{\mu_w B_w} \Delta_y \Phi_w \right] \Delta y_{i,j,k} = \left( T_{wy,i+1/2,j,k} \Phi_{wy,i+1,j,k} - \Phi_{wy,i,j,k-1} \right)
\]

And the approximation in the z-direction may be expressed as:

\[ \frac{\partial}{\partial z} \left[ \frac{A_w C_w K_w}{\mu_w B_w} \frac{\partial \Phi_w}{\partial z} \right] \Delta z_{i,j,k} = \left( T_{wz,i,j+1/2,k} \Phi_{wz,i,j+1,k} - \Phi_{wz,i,j-1,k} \right)
\]

Let

\[ \Delta \left[ \frac{\rho_w}{\rho_w} (1 - \Phi) \frac{\partial C_w}{\partial t} \right] \Delta x_{i,j,k} \equiv \Delta_x \left[ \frac{A_w C_w K_w}{\mu_w B_w} \Delta_x \Phi_w \right] \Delta x_{i,j,k}
\]

Substituting (7), (9) and (11) gives:

\[ \Delta \left[ \frac{\rho_w}{\rho_w} (1 - \Phi) \frac{\partial C_w}{\partial t} \right] \Delta x_{i,j,k} = \left( \frac{C_w}{\rho_w} \right)_{i,j,k} \frac{\partial C_w}{\partial t} \Delta x_{i,j,k}
\]

\[ \left( \frac{T_{wx,i+1/2,j,k} \Phi_{wx,i+1,j,k} - \Phi_{wx,i,j,k-1}}{\rho_w} \right)
\]

where

\[ \Phi_{wx,i,j,k} \] water potential and is defined as:

\[ \Phi_{wx} = \Delta P_w - \gamma_w \Delta z
\]

and

\[ \Delta \Phi_w = \Delta P_w - \Delta P_{cow} - \gamma_w \Delta z
\]

Substituting (16) and rearranging (13) gives:

\[ \Delta \left[ \frac{\rho_w}{\rho_w} (1 - \Phi) \frac{\partial C_w}{\partial t} \right] \Delta x_{i,j,k} = \left( \frac{C_w}{\rho_w} \right)_{i,j,k} \frac{\partial C_w}{\partial t} \Delta x_{i,j,k}
\]

\[ \left( \frac{T_{wx,i+1/2,j,k} \Phi_{wx,i+1,j,k} - \Phi_{wx,i,j,k-1}}{\rho_w} \right)
\]

\[ \left( \frac{T_{wy,i+1/2,j,k} \Phi_{wy,i+1,j,k} - \Phi_{wy,i,j,k-1}}{\rho_w} \right)
\]

\[ \left( \frac{T_{wz,i,j+1/2,k} \Phi_{wz,i,j+1,k} - \Phi_{wz,i,j-1,k}}{\rho_w} \right)
\]

\[ \left( \frac{T_{wx,i+1/2,j,k} \Phi_{wx,i+1,j,k} - \Phi_{wx,i,j,k-1}}{\rho_w} \right)
\]

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\[ \varphi_y = \{(n - n_x), (n + n_x)\} \quad \text{for} \quad n_x = 1 \]

\[ \varphi_y = \{(n - n_y, n + n_y)\} \quad \text{for} \quad n_y = 1 \]  

(29)

The variable \( \varphi \) is the value of \( \varphi \) at gridblock \( \Psi \). The coefficient \( (T_wC_{w_n})_{i,j,k,m} \) is the \( (T_wC_{w_n}) \) interactions between gridblock \( (i,j,k) \) and its immediate neighboring gridblock \( \Psi_{i,j,k} \).

Using this notation, the implicit finite difference equation in Gridblock \( n \), in multidimensional space becomes:

\[ \sum_{m\in\Psi_n} (T_w^{n+1}C_{w_n}^{n+1})_{i,j,k,m} \left( \Delta_m p_{n+1}^{m} - \Delta_m p_{n}^{m} - \psi_{m,n}^{n} \Delta_m x^{n+1} \right) = -\psi_{m,n}^{n} p_{-1}^{m} + N_{w,n} \Delta z_{w} + N_{n} \Delta C_{w_n} \]  

(30)

2.1.3 Bacteria Transport Equation

The bacteria transport equation is given as:

\[ \Delta \left[ \frac{C_w}{\mu_{w} \rho_{w}} \phi \Delta \Phi_w \right] + q_{w} C_{w} = \delta \left( \phi S_{w} \rho_{w} \Phi_w + \sigma \right) \]  

(31)

In three-dimensional coordinates:

\[ \frac{\partial}{\partial x} \left[ \frac{C_w}{\mu_{w} \rho_{w}} \phi \right] \delta x + \frac{\partial}{\partial y} \left[ \frac{C_w}{\mu_{w} \rho_{w}} \phi \right] \delta y + \frac{\partial}{\partial z} \left[ \frac{C_w}{\mu_{w} \rho_{w}} \phi \right] \delta z + q_{w} C_{w} = \delta \left( \phi S_{w} \rho_{w} \Phi_w + \sigma \right) \]  

(32)

Applying a similar finite difference approximation to that performed on the nutrient transport equation, the bacteria transport equation given above can be written as:

\[ \sum_{m\in\Psi_n} (T_w^{n+1}C_{w_n}^{n+1})_{i,j,k,m} \left( \Delta_m p_{n+1}^{m} - \Delta_m p_{n}^{m} - \psi_{m,n}^{n} \Delta_m x^{n+1} \right) = -\psi_{m,n}^{n} p_{-1}^{m} + B_{w,n} \Delta A_{w_n} + B_{z,n} \Delta S_{w} + \Delta z_{w} \]  

(33)

where

\[ B_{p} = \frac{1}{\Delta \left[ \frac{C_w}{\mu_{w} \rho_{w}} \phi \right]} \delta x + \frac{\partial}{\partial \delta z} \left[ \frac{C_w}{\mu_{w} \rho_{w}} \phi \right] \delta z + q_{w} C_{w} = \delta \left( \phi S_{w} \rho_{w} \Phi_w + \sigma \right) \]  

(34)

2.1.4 Multiphase Flow Equations

The model equations for oil, water and gas are given as follows.

2.1.4.1 Oil

The model equation for oil is given as:

\[ \frac{\partial}{\partial x} \left[ \frac{A_{k} C_{w_n}}{\mu_{w} \rho_{w}} \frac{\delta \phi}{\delta x} \frac{\delta \phi}{\delta x} \right] + \frac{\partial}{\partial y} \left[ \frac{A_{k} C_{w_n}}{\mu_{w} \rho_{w}} \frac{\delta \phi}{\delta y} \frac{\delta \phi}{\delta y} \right] + \frac{\partial}{\partial z} \left[ \frac{A_{k} C_{w_n}}{\mu_{w} \rho_{w}} \frac{\delta \phi}{\delta z} \frac{\delta \phi}{\delta z} \right] + q_{o} = v_{b} \frac{\partial}{\partial x} \frac{A_{k} C_{w_n}}{\mu_{w} \rho_{w}} \phi \]  

(35)

Applying the finite difference approximation to the second order partial differential term in the \( x \), and \( z \) directions at gridpoints \( (i,j,k) \) and simplifying results in:

\[ \Delta \left[p_{x}^1 \Phi_{x}^1 \right] = T_{o x} \left( \phi_{x+1,i,j} - \phi_{x,i,j} \right) \]

(36)

where

\[ T_{o} = \frac{k_{o w} A_{k}}{\mu_{w} \rho_{w}} \]  

(37)

From the definition of change in fluid potential, the oil equation becomes:
\[ \Delta \left[ AK_{nw} \left( \Delta P - \gamma_{o} \Delta Z \right) \right]_{i,j,k}^{n+1} = (C_{wp} \Delta t_{P} + C_{uw} \Delta t_{S} + C_{cow} \Delta t_{S} - \Delta q_{osc}^{n+1}) \]

Where

\[
C_{wp} = \frac{1}{\Delta t} \left( \phi + \phi_{n+1} \left( \frac{1}{B_w^o} \right) \right) - S_{w}^{o}
\]

\[
C_{uw} = \frac{1}{\Delta t} \left( \phi_{n+1} \left( \frac{1}{B_w^o} \right) \right)
\]

Writing (38) in a control volume finite difference (CVFD) method, the final oil equation becomes:

\[
\sum_{m \in \omega_a} T_{m}^{n+1} \left( \Delta m_{u}^{n+1} - \gamma_{a,m}^{n} \Delta m_{z}^{n} \right) = C_{wp} \Delta t_{P} + C_{uw} \Delta t_{S} + C_{cow} \Delta t_{S} - \Delta q_{osc}^{n+1}
\]

2.1.4.2 Water

The model equation for water is given as:

\[
\frac{\partial}{\partial x} \left[ \frac{AK_{nw}}{B_w^o} \phi_{w} \frac{\partial \phi_{w}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{AK_{nw}}{B_w^o} \phi_{w} \frac{\partial \phi_{w}}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{AK_{nw}}{B_w^o} \phi_{w} \frac{\partial \phi_{w}}{\partial z} \right] = \frac{\partial}{\partial t} \phi_{w} + \frac{\partial}{\partial t} \phi_{w}^{n+1} + \frac{1}{B_w^o} (1 - S_{w}^{o} - S_{w}^{n+1})
\]

Writing (45) in a control volume finite difference (CVFD) method, the final water equation becomes:

\[
\sum_{m \in \omega_a} T_{m}^{n+1} \left( \Delta m_{u}^{n+1} + \Delta m_{c}^{n+1} - \gamma_{a,m}^{n} \Delta m_{z}^{n} \right)
\]

\[
= C_{wp} \Delta t_{P} + C_{uw} \Delta t_{S} + C_{cow} \Delta t_{S} - \Delta q_{osc}^{n+1}
\]

2.2 Experimental Data

The pressure response from the simulation was observed based on the experimental parameters obtained from the core flood test. In the experiment, 0.1 pore volume of microbial solution was introduced into the core. This was followed by the injection of 0.2 pore volume of nutrient solution and incubated for 2 days. After the incubation period, 0.2 pore volume of nutrient solution was injected after which the core was waterflooded at 1.5 cm³/min. The values of the parameters used in the simulation are listed in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td></td>
</tr>
<tr>
<td>Length, cm</td>
<td>14</td>
</tr>
<tr>
<td>Width, cm</td>
<td>14</td>
</tr>
<tr>
<td>Thickness, cm</td>
<td>9</td>
</tr>
<tr>
<td>Porosity, %</td>
<td>15</td>
</tr>
<tr>
<td>Permeability [k_s,k_i,k_o] md</td>
<td>410, 410, 200</td>
</tr>
<tr>
<td>Grid size (Ax, Ay, Az), cm</td>
<td>1.1, 1, 1</td>
</tr>
<tr>
<td>Number of grid (x,y,z)</td>
<td>14, 14, 9</td>
</tr>
<tr>
<td>Fluid</td>
<td></td>
</tr>
<tr>
<td>Initial oil saturation, %</td>
<td>87</td>
</tr>
<tr>
<td>Residual water saturation, %</td>
<td>13</td>
</tr>
<tr>
<td>Initial oil viscosity, cp</td>
<td>19</td>
</tr>
</tbody>
</table>
Oil compressibility, psi⁻¹ 0.00014
Initial pressure, psi 28
Water injection rate, cm³/min 1.5
Oil production rate, cm³/min 2
Water production rate, cm³/min 0.53
Gas production rate, cm³/min 1.0

Bacteria and nutrient
Bacteria injection rate, cm³/min 0.5
Injected microbial conc. % pore volume 0.1
Injected nutrient conc. % pore volume 0.2
Diffusion coefficient for bacteria, cm³/day 5.1
Diffusion coefficient for nutrients, cm³/day 7.4
Maximum growth rate, day⁻¹ 5.3
Decay rate, day⁻¹ 34
Nutrient solution
Beef extract, g/c 1.0
Yeast extract, g/c 2.0
Peptone, g/c 5.0
Sodium Chloride, g/c 5.0

3 RESULTS
The simulator used in this project is known as UTCHEM [14]. UTCHEM is a three dimensional chemical flooding simulator that incorporates the MEOR model. The solution scheme is analogous to the implicit pressure explicit hrs saturation (IMPES) where the pressure was solved implicitly and saturations were solved explicitly. A time step of 2 hours was considered and a no-flow boundary of the core was implemented by setting pressure gradient at the boundary interfaces to zero.

TABLE 2
PRESSURE AND PRESSURE DROP RESPONSE DURING CONTINUOUS WATERFLOODING

<table>
<thead>
<tr>
<th>Pressure, P (psi)</th>
<th>Pressure drop ∆P = (P₋Pᵢ), psi</th>
<th>Time step ∆t, hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>36.00</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>43.00</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>45.50</td>
<td>17.5</td>
<td>6</td>
</tr>
<tr>
<td>49.30</td>
<td>21.3</td>
<td>8</td>
</tr>
<tr>
<td>50.60</td>
<td>22.6</td>
<td>10</td>
</tr>
<tr>
<td>52.20</td>
<td>24.2</td>
<td>12</td>
</tr>
<tr>
<td>49.30</td>
<td>21.3</td>
<td>14</td>
</tr>
<tr>
<td>47.00</td>
<td>19.0</td>
<td>16</td>
</tr>
<tr>
<td>48.00</td>
<td>20.0</td>
<td>18</td>
</tr>
<tr>
<td>46.80</td>
<td>18.8</td>
<td>20</td>
</tr>
<tr>
<td>45.70</td>
<td>17.7</td>
<td>22</td>
</tr>
<tr>
<td>45.60</td>
<td>17.6</td>
<td>24</td>
</tr>
<tr>
<td>44.50</td>
<td>16.5</td>
<td>26</td>
</tr>
<tr>
<td>43.70</td>
<td>15.7</td>
<td>28</td>
</tr>
</tbody>
</table>

The graphical plots of the simulation result are presented below.

Fig.1. Pressure in core with time

Fig. 2. Pressure drop in core with time

The simulation results showed that the sharp increase of pressure after the microbial flooding is caused by pore plugging and the production of biogenic gas. The pore plugging reduces the rock permeability and the increase of gas saturation decreases the relative permeability of the oil and water phase. As the time increases and water flooding proceeds, the permeability recovered slightly with the declogging of microbes in the pore space. Consequently, a decrease in pressure drop was observed from the core flood.

4 CONCLUSION
Numerical models of pressure transient response during MEOR have been presented. It was a logical step to incorpo-
rate a MEOR model into an existing chemical flooding simulator. The simulator was run based on the experimental data and in a three dimensional framework to determine the effects of pressure drop as water flooding continued, describing the transient response of the core during the MEOR process. In addition, UTCHEM can accommodate a number of substrates accepters, biotic decay, biomass growth, and biomass attachment which are all microbial enhanced oil recovery processes. Since MEOR reduces or eliminates the need to use harsh chemicals during oil recovery, it is an environmentally friendly and economically feasible method of carrying out enhanced oil recovery, amidst recent growing concerns of certain methods like fracturing. Therefore, there is need for special skills and logical steps to incorporate MEOR into a simulator that will describe or analyze the transient response of a reservoir during microbial enhanced oil recovery processes.

NOMENCLATURE

\[ A = \text{Cross sectional area, ft}^2 \]
\[ A_x, A_y, A_z = \text{Cross sectional area normal to the x, y and z directions, ft}^2 \]
\[ B = \text{Formation volume factor (FVF), bbl/STB} \]
\[ C_{en} = \text{Concentration of nutrient in the injected water, lb/ft}^3 \]
\[ C_{eb} = \text{Concentration of bacteria in the injected water, lb/ft}^3 \]
\[ K_r = \text{Relative permeability to gas, dimensionless} \]
\[ K_{x,y,z} = \text{Permeability in the x, y and z directions respectively, mD} \]
\[ m = \text{a member of gridblock set } \psi_n \]
\[ n = \text{number of gridblocks} \]
\[ n_x, n_y, n_z = \text{number of gridblocks in the x, y and z directions} \]
\[ P = \text{Pressure, psi} \]
\[ P_{on} = \text{Oil pressure for gridblock n, psi} \]
\[ P_{co} = \text{Gas/Oil capillary pressure, psi} \]
\[ P_{cow} = \text{Oil/water capillary pressure, psi} \]
\[ q = \text{production rate, scf/D} \]
\[ R_{on} = \text{Oil residual in gridblock n, STB/D} \]
\[ R_s = \text{SolutionGOR, scf/STB} \]
\[ S = \text{saturation, fraction} \]
\[ t = \text{time, days} \]
\[ \Delta t = \text{time step (} = t^{n+1} - t^n \text{), days} \]
\[ U_l = \text{Darcy flux} \]
\[ \gamma = \text{gravity, psi/ft} \]
\[ \Delta = \text{difference operator} \]
\[ \Delta_n = \text{difference operator in the gridblock domain} \]
\[ \Delta_t = \text{difference operator in the time domain} \]
\[ \mu = \text{viscosity, cp} \]
\[ p = \text{density, lb/ft}^3 \]
\[ T_p = \text{Transmissibility} \]
\[ \phi = \text{Fluid potential, psi} \]
\[ \phi_p = \text{porosity, fraction} \]
\[ \psi_n = \text{set of gridblocks associated with but excluding gridblock n in natural ordering} \]

Subscripts

\[ g = \text{gas} \]
\[ a = \text{oil} \]
\[ w = \text{water} \]
\[ p = \text{phase} \]
\[ i = \text{index for blocks in the x direction} \]
\[ j = \text{index for blocks in the y direction} \]
\[ m = \text{neighbouring gridblock to gridblock n} \]
\[ n = \text{gridblock} \]
\[ sc = \text{standard conditions} \]
\[ t = \text{time} \]

REFERENCES
