ISSN 2347-4289

Combined Control Scheme For Monitoring Quality Characteristics

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ABSTRACT: In the literature, the Exponentially Weighted Moving Average (EWMA) and Exponentially Weighted Moving Variance (EMWV) control schemes have been used separately to monitor the process average and process variability respectively. Here the two are combined and applied on simulated process with different level of variation. The control limit interval (CLI) and the average run length (ARL) were evaluated for the combined chart. The combined chart performed better than the two independently. Furthermore, an algorithm was developed for the two control charts and implemented on visual basic VB6.0. The obtained results show that the combined EWMA and EWMV control chart is very sensitive in detecting shift in production process and every shift in the process mean is always preceded by shift in the process variability.

Keywords: Statistical process control, control chart, Exponentially Weighted Moving Average (EWMA), Exponentially Weighted Moving Variance (EMWV)

1 INTRODUCTION

Statistical Process Control (SPC) scheme are extensively used to detect process excursions and thus prevent the production of defective products as quality of products today is in the centre of attention worldwide. There are several forms of process variation, but process means shifts are the primary focus of most control chart design. The Shewhart control chart (e.g. $\overline{X}$, R, P, C-chart e.t.c) so named after the pioneering work of Dr Walter Shewhart, have wide acceptability in industries, it can be shown that, if there are sharp, intermittent changes in a process, these types of charts are highly effective in detecting them. However, if one is interested in small persistent shift in a process, other types of control chart may be preferred, for example, Exponentially Weighted Moving Average (EWMA) control charts originally described by Robert(1959), and Exponentially Weighted Moving Variance (EMWV) control charts as advocated by Montgomery and Mastrangelo(1991). EWMA and EWMV charts are becoming much better popular among the practitioners because of their superior ability to detect small process shift (Narinderjit Singh 2006). The EWMA and EWMV procedures will usually give tighter process control than the classical quality control schemes such as Shewhart schemes, because EWMA and EWMV procedures give an early indication of process changes, this is consistent with a management philosophy that encourages doing it right the first time(Adekeye2009). An exponentially weighted moving average is a moving average of past data where each data point is assigned a weight. These weights decrease in an exponentially decaying fashion from present into the remote past, thus the moving average tends to be a reflection of the more recent process performance, because most of the weight is allocated to the most recently collected data. The amount of decrease of the weights is an exponential function of the weighting factor, (λ) which can assume values between 0 and 1. Exponentially weighted moving variance is a moving variance of past data that is equally having a weight attached. Exponentially weighted moving averages will gradually depending on the weighting factor, move to the new mean of the process if a shift in the mean occurs, while the exponentially weighted moving variances will remain unchanged. If there is a shift in the process variability, the exponentially weighted moving variances will gradually move to the new level while the exponentially weighted moving averages still vary about the process mean. The Shewhart control schemes for mean and range provide information on variability of the quality characteristics, consistency of performance and the mean of the quality characteristics, however Shewhart control scheme are only efficient in detecting large shift value in process mean and process variability. The (EWMA) control schemes can be designed to quickly detect small shifts in the mean of process, (Macgregor and Harris 1990), and exponentially weighted moving variance (EWMV) control scheme is very efficient in detecting small shift in process variability. A combined EWMA & EWMV gives improved properties when shift in both process mean and process variability are to be detected. This work focus on combining Exponentially Weighted Moving Average (EWMA) control scheme and Exponentially Weighted Moving Variance (EWMV) control scheme to detect shifts in the mean and the variance of a simulated process.

2. Materials and Methods

Formulation of the EWMA and the EWMV charts.
For sequentially recorded observations, which can either be individually observed values (X_t) from the process or sample averages ($\overline{X}_t$), the formulation of the EWMA and the EWMV charts are given below. The EWMA chart:

$$Z_t = \lambda Q_t + (1-\lambda) Z_{t-1}\,,\; 0 < \lambda \leq 1 \,, t=1,2,\ldots,n \qquad (1.0)$$

$$CL = \overline{X}$$

$$LCL = \overline{X} - 3\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} \qquad (2.0)$$

$$UCL = \overline{X} + 3\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} \qquad (3.0)$$

Where: Z_t is the value plotted on the control chart and is a weighted average of all previous plotted values Z_0 is the es-

ISSN 2347-4289

timated process mean and a starting value for the EWMA λ is a smoothing parameter Q_t is the sequentially recorded observations (which can either be individually observed values (X_t) from the process or sample averages ($\bar{X}_t$) obtained from a designated sampling plan). σ is the standard deviation of the observations The EWMV chart:

$$V_t^2 = \lambda(Q_t - Z_t)^2 + (1-\lambda)V^2_{t-1}, 0 \le \lambda \le 1, t = 1,2,..n \quad (4.0)$$

$$CL \quad \sigma^2 \quad (5.0)$$

$$LCL = \sigma^2 - 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (6.0)$$

$$UCL = \sigma^2 + 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (7.0)$$

Where:

V^2_t is exponentially weighted moving variance

V^2_0 is the variance of the individually observed values or the sample averages. Q_t is the sequentially recorded observations (which can either be individually observed values (X_t) from the process or sample averages ($\bar{X}_t$) obtained from a designated sampling plan).

Z_t is the corresponding EWMA value λ is EWMV weighting parameter An algorithm is developed to compute EWMA and EWMV statistics from the observed values or the sample averages and using the algorithm, a source code for computing EWMA and EWMV statistic is written in VB6.0 and used to compute the statistics taking into consideration the procedures described above.

The algorithm is as follows:

1.1 Set counter = 0
1.2 Set sum of X = 0
1.3 Set mean of X = 0
2 Open “data.txt”
3 Loop while end of file “data.txt” = false
3.1 Set counter = counter + 1
3.2 Read X from file “data.txt”
3.3 Array Data (counter) = X
4 Close file (data.txt)
5 Set counter = 0
6 Loop while counter ≤ upper bound of ArrayData
6.1 Set counter = counter +1
6.2 Set sum of X = sum of X + ArrayData(counter)
7 Mean of X = sum of X/counter
8 Set counter = 0
9 Set sum of X = 0
10 Loop while counter ≤ upper bound of ArrayData
10.1 Set counter = counter + 1
10.2 Set Sum of X = Sum of X + (ArrayData(counter) – Mean of X)**2
11 End loop
12 Set V_0 = Sum of X/(counter -1)
13 Set Z_0 = Mean of X
14 Open “ComputedData.txt”
15 Set Counter = 0
16 Loop while counter ≤ upper-bound of ArrayData
16.1 counter = counter + 1
16.2 EWMA = lamda*ArrayData(counter) + (1-lamda)*Z_0
16.3 EWMV = lamda*(ArrayData(counter)-EWMA)^2 + (1-lamda)*V_0
16.4 Write counter, ArrayData(counter), EWMA, EWMV into file “ComputedData.txt”
16.5 Set Z_0 = EWMA
16.6 Set V_0 = EWMV
17 End loop
18 Close file “ComputedData.txt”
19 Stop.

2.1.1 Choosing The Value Of The Weight Parameter (λ)

Various approaches have been proposed for choosing the value of λ. Lucas and Saccucci (1990), states that the choice “can be left to the judgment of the quality control analyst” and points out that the smaller the value of λ “the greater the influence of the historical data”. Furthermore, smaller values of λ should be used if early recognition of smaller shifts is desired. If λ = 1, the EWMA control chart degrades to the usual Shewhart $\bar{X}$ chart. Thus the larger the value of λ the more the weight assigned to the recent data and the shallow the memory of the EWMA. The smaller the value of λ, the more the weights given to older data and the deeper the memory of the EWMA. In order to see the influence of the weighting factor on the control charts, different value λ was applied on the simulated process in this study.

2.1.2 Simulation Process

Simulation is the use of mathematical model to recreate a situation, often repeatedly, so that the likelihood of various outcomes can be more accurately estimated. Simulation has also been defined as the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation by Gupta and Hira(2012). We simulate a process that is normally distributed; the model is given as;

$$f_{(x)} = f_{(x:\mu,\sigma^2)} = \frac{1}{\sqrt{2\pi\sigma^2}}\,\ell^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}, -\infty \le x \le \infty, \mu \ge 0, \sigma \ge 0 \quad (8)$$

Three different levels are considered for the variance σ^2 ;

(i) Small variation: σ^2 = 0.268
(ii) Medium variation: σ^2 = 2.144
(iii) Large variation: σ^2 = 4.288
and μ = 75 is set to simulate 200 cases.

2.1.3 Computation of ARL

The average run length (ARL) at a given level is the average number of samples taken before an action signal is given. If the process monitoring scheme employs only the single alarm rule “signal the first time that a point Q plots outside control limits,” and the process is stable , the values Q1, Q2, Q3,…. .can be modeled as random draws from a fixed distribution(Stephen 2011), then using the notation q = P(Q1 plots outside control limits)

and ARL = $\frac{1}{q}$

2.1.4 CONTROL LIMIT INTERVAL (CLI)
The control limit interval (CLI) is the difference between the upper control limit and the lower control limit. The lower the value of CLI the better the performance of the control scheme (Adekeye 2012)

The CLI for EWMA is

CLI_{EWMA} = UCL – LCL

$$= (\overline{X} + 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}) - (\overline{X} - 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}})$$

$$= 6\sigma\sqrt{\frac{\lambda}{2-\lambda}}$$

And the CLI for EWMV is

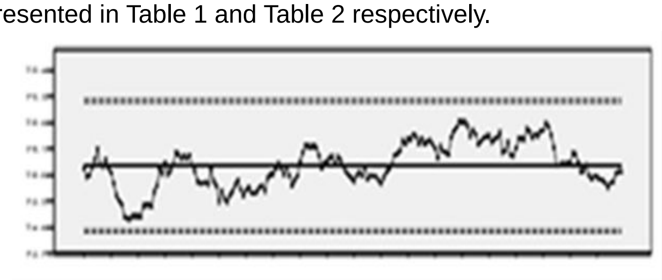
CLI_{EWMV} = UCL – LCL

$$(\sigma^2 + 3\sigma\sqrt{\frac{\lambda}{2-\lambda}}) - (\sigma^2 - 3\sigma\sqrt{\frac{\lambda}{2-\lambda}})$$

$$= 6\sigma\sqrt{\frac{\lambda}{2-\lambda}}.$$

3. Results and discussion

The EWMA &EWMV control charts are presented below in Figure 1 to Figure 6 for the simulated process with small variation, Figure 7 to Figure 12 for the simulated process with medium variation and Figure 13 to Figure 18 for the simulated process with large variation. The Average Run Length (ARL) and the Control Limit Interval (CLI) were computed for EWMA and EWMV control charts. The summary of CLI and ARL are presented in Table 1 and Table 2 respectively.



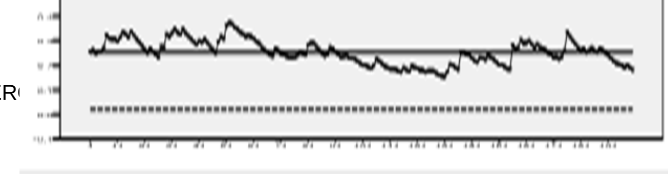


Figure 1: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH SMALL VARIATION (Var = 0.268 & λ = 0.05)

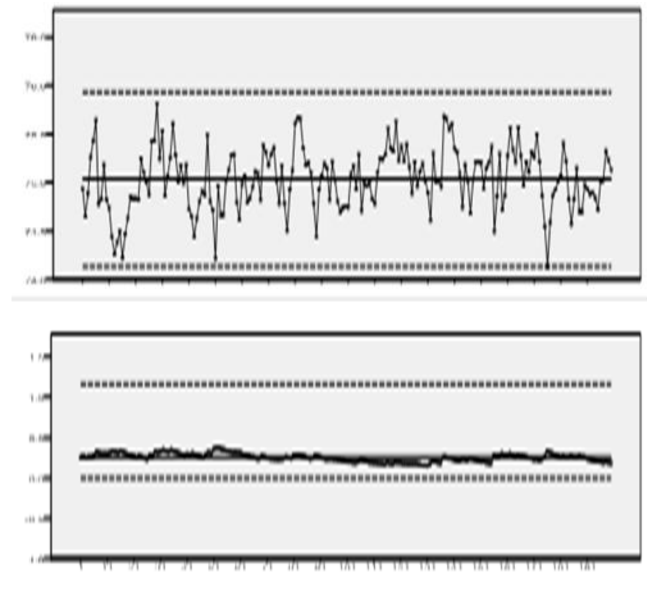
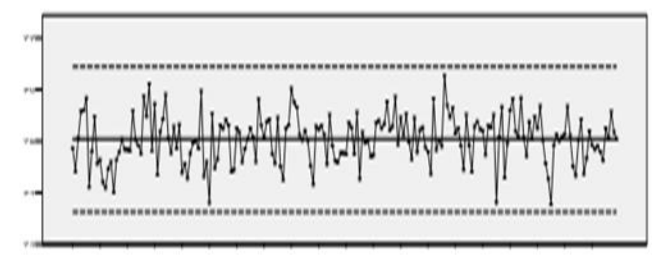


Figure 2: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH SMALL VARIATION (Var = 0.268 & λ = 0.5)



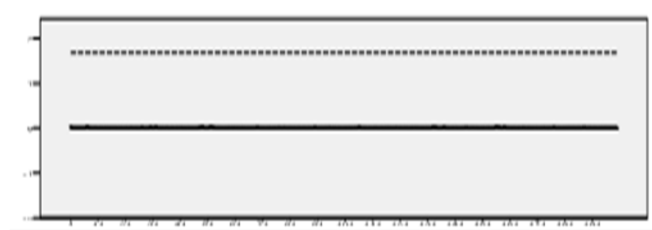


Figure 3: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH SMALL VARIATION (Var = 0.268 & λ = 0.9)

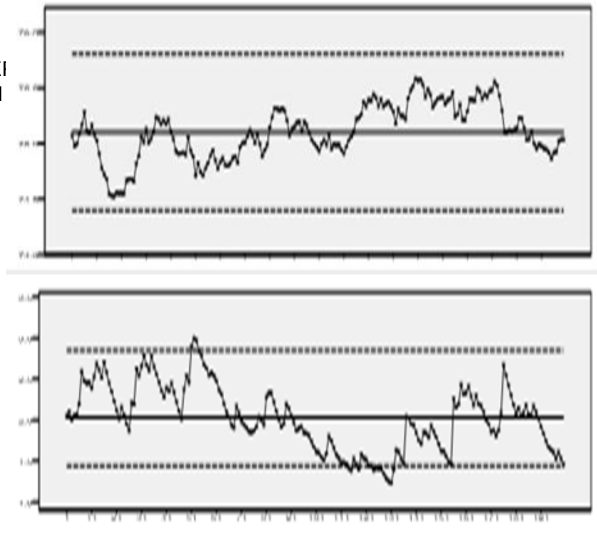


Figure 4: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH MEDIUM VARIATION (Var = 2.144 & λ = 0.05

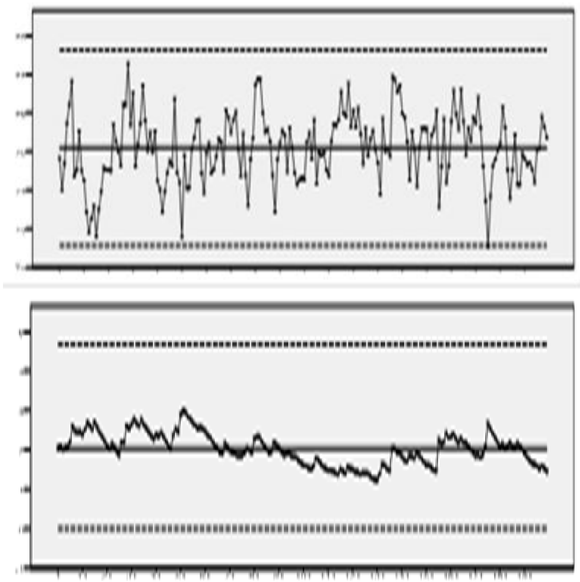


Figure 5: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH MEDIUM VARIATION (Var = 2.144 & λ = 0.5)

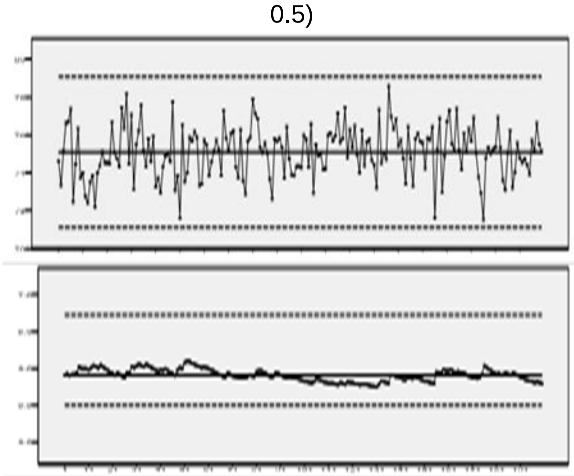


Figure 6: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH LARGE VARIATION (Var = 4.288 & λ = 0.05)

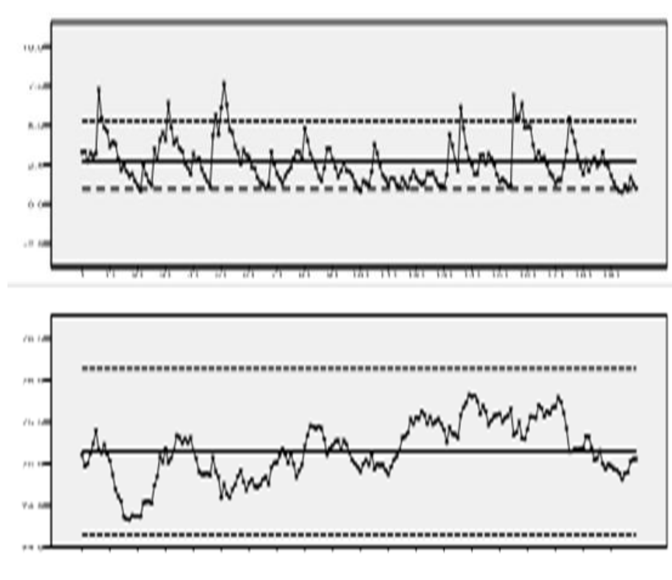


Figure 7: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH LARGE VARIATION (Var = 4.288 & λ = 0.05)

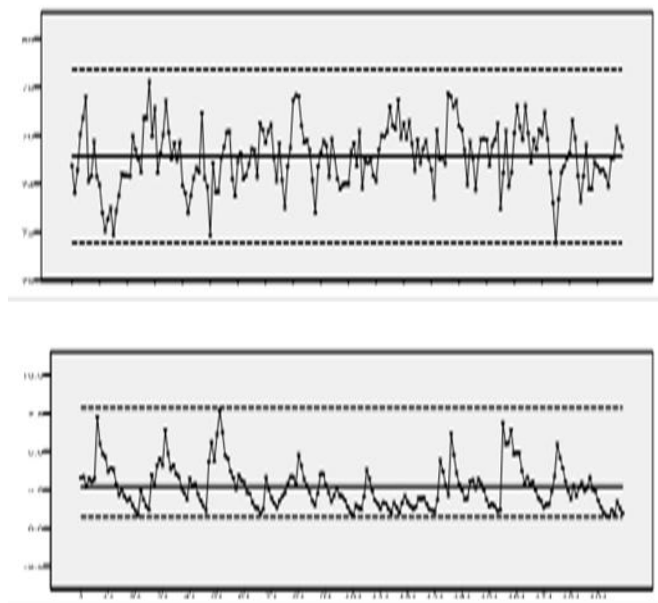
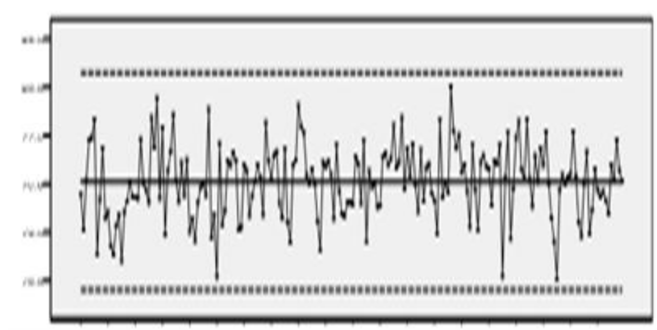


Figure 8: EWMA &EWMV CONTROL CHARTS FOR SIMULATED DATA WITH LARGE VARIATION (Var = 4.288 & λ = 0.5)



ISSN 2347-4289

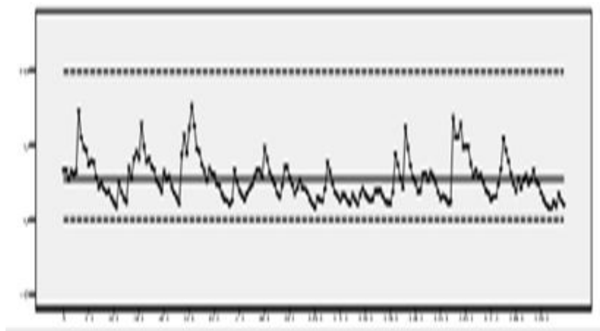


Figure 9: EWMA & EWMV CONTROL CHARTS FOR SIMULATED DATA WITH LARGE VARIATION (Var = 4.288 & λ = 0.9)

Table 1: CLI for EWMA&EWMV control chart

		λ					
		0.050	0.100	0.250	0.500	0.750	0.900
Σ	0.518	0.500	0.715	1.175	1.793	2.409	2.810
	1.464	1.388	2.020	3.320	5.068	6.808	7.940
	2.071	2.000	2.858	4.697	7.170	7.456	11.233

Source: Simulation output.

The values in Table 1 shows the various value of CLI for the selected values of the weighting parameter (λ) for small variation, medium variation and large variation. The nomogram of which is presented in Figure 10.

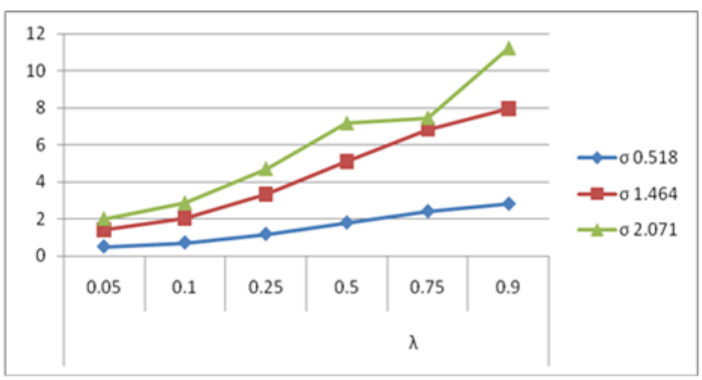


Figure 10 : CLI curve for EWMA&EWMV chart

Table 2: Average Run Length (ARL) for EWMA control chart

	Λ					
	0.05	0.10	0.25	0.50	0.75	0.90
ARL	2	2	4	12	49	147

Source: Simulation output

The values in Table 2 shows the various value of ARL for the selected values of the weighting parameter (λ), the nomogram of which is presented in Figure 11.

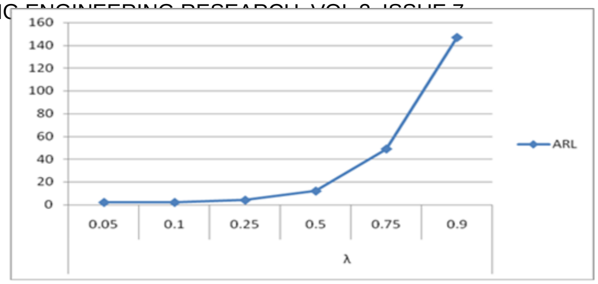


Figure 11: ARL curve for EWMA control chart

The results of the application of the combined EWMA & EWMV control scheme on the simulated process shows that shift in the process average is always preceded by a shift in the process variability as evident in the combined charts presented above and generally speaking, smaller value of the weighting factor seems more sensitive in recognizing shift in simulated process quality characteristics. The result on Table 2 shows that the CLI for small value of weighting factor (λ) is lower than that of high value of the weighting parameter. The results however show that the CLI for EWMA and EWMV chart are influenced by the size of variation and we conclude that small value of the weighting parameter (λ) on simulated process with small variation produces small value of CLI. The result on Table 1 shows that the ARL for small value of weighting factor (λ) is lower than that of high value of the weighting parameter and the curve of the ARL for EWMA in Figure 20 shows that EWMA chart is more sensitive to shift when the weighting parameter is small. In combined EWMA&EWMV control scheme, the process is adjudged to be stable if both the process average and variability is stable, if any of the two is out of control, the process is considered to be out of control.

3.1 Conclusion

The Combined EWMA & EWMV control scheme has been proposed as alternative means of monitoring process variation, analyzed by varying the weighting factor. The results show that the combined EWMA & EWMV control chart is very sensitive to small shift in the process mean and process variability.

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