A Comparative Study Of Various Sunshine Based Models For Estimating Global Solar Radiation In Zaria, North-Western, Nigeria

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ABSTRACT: Solar radiation is a primary driver of many physical, chemical and biological processes on the earth’s surface. In solar applications, one of the most important parameters needed is the long-term average daily global solar radiation. For locations where no actual measured values are available, a common practice is to estimate average daily global solar radiation using appropriate empirical correlations based on available measured meteorological data at that location. In this regard, twelve (12) different empirical models based on Angstrom-Prescott model were selected to estimate the monthly average daily global solar radiation on a horizontal surface for Zaria, Kaduna State, North-Western, Nigeria (Latitude 11.06°N, Longitude 07.41°E and altitude 110.9 m above sea level) during the period of thirty one years (1980 – 2010) using the measured global solar radiation and sunshine hour. The twelve (12) selected models were compared on the basis of the statistical error tests: Mean Bias Error (MBE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), t – test, and coefficient of correlation (R). Based on the MBE and t – test equation 16 is extremely recommended. Based on RMSE equation 15 is extremely recommended and based on MPE equation 13 is extremely recommended. The newly developed regression equations (16, 15 and 13) for estimating global solar radiation in Zaria-Nigeria are based on the modified sunshine based models of the Ampratwum and Dorvio, Newland and Ogelman et al. models and are also applicable in estimating global solar radiation in regions with similar climatic information where solar radiation data are not available.

Keywords: global solar radiation, sunshine hours, statistical test, Angstrom model, Zaria-Nigeria.

1 INTRODUCTION
Solar radiation arriving on earth is the most fundamental renewable energy source in nature. Solar energy, radiant light and heat from the sun, has been harnessed by humans since ancient times using a range of ever evolving technologies. The energy from the sun could play a key role in de-carbonizing the global economy alongside improvements in energy efficiency and imposing costs on greenhouse gas emitters. In the studies of solar energy, solar radiation data and other meteorological parameters at a given location are fundamental input for solar energy applications such as photovoltaic, solar-thermal systems, solar furnaces and passive solar design [7]. The data should be reliable and readily available for design, optimization and performance evaluation of solar systems for any particular location [7]. The best way to determine the amount of global solar radiation at any location is to install measuring instruments such as pyranometer and pyrheliometer at that particular place and to monitor and record its day-to-day recording, which is really a very tedious and costly exercise [15].

Compared to measurements of other meteorological variables, the measurement of solar radiation is more prone to errors and often encounters more problems such as technical failure and operation related problems. These problems could be one of many: calibration problems, problems with dirt on the sensors, accumulated water, shading of sensor by masts, etc [3]. The measurement of solar radiation is not easily available in many developing countries as there are few meteorological stations measuring the solar radiation data due to the incapability to afford the measuring equipment and techniques involved [18].

It is therefore important to consider methods of estimating the global solar radiation based on the readily available meteorological parameters. Several empirical models have been proposed to calculate the global solar radiation, using available meteorological, climatological and geographical parameters such as sunshine duration, maximum and minimum temperatures, cloud cover, wind speed, latitude, relative humidity, precipitation and rainfall. The most commonly used parameter for estimating global solar radiation is sunshine hours [1], [4], [6], [17], [20 – 21] and [26]. This is because sunshine hours can be easily and reliably measured and the data are widely available. The most widely used method for estimating global solar radiation using sunshine hour is that of Angstrom [16].

The objective of this study was to evaluate various empirical models for the prediction of monthly average daily global solar radiation on a horizontal surface from sunshine duration in Zaria-Nigeria and to select the most adequate model based on the statistical test analysis subjected to.

2 METHODOLOGY
The measured monthly average daily global solar radiation and sunshine hour covering a period of thirty one years (1980 – 2010) for Zaria, North – Western, Nigeria was obtained from the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos, Nigeria. Monthly averages over the thirty one years of the data in preparation for correlation are presented in Table 2. The first correlation proposed for estimating the monthly average global solar radiation is based on the method of [5]. The original Angstrom- Prescott type regression equation-related monthly average daily radiation to clear day radiation in a given location and average fraction of possible sunshine hours is given by the equation:

\[
\frac{H}{H_o} = a + b \left( \frac{s}{S_o} \right)
\]

where \(H\) is the monthly average daily global solar radiation on a horizontal surface (MJ/m²/day), \(H_o\) is the monthly average daily extraterrestrial radiation on a horizontal surface (MJ/m²/day), \(s\) is the monthly average daily hours of bright sunshine, \(S_o\) is the monthly average day length and \(a\) and \(b\) values are the Angstrom empirical constants. The monthly
average daily extraterrestrial radiation on a horizontal surface \( (H_o) \) can be calculated for days giving average of each month [14], [25] and [27] from the following equation [14] and [27]:

\[
H_o = \left( \frac{24}{\pi} \right) I_{sc} \left[ 1 + 0.033 \cos \left( \frac{360n}{365} \right) \right] \cos \phi \cos \delta \sin \delta W_s + \left( \frac{2nW_s}{360} \right) \sin \phi \sin \delta
\]

(2)

where \( I_{sc} \) is the solar constant (=1367 Wm\(^{-2}\)), \( \phi \) is the latitude of the site, \( \delta \) is the solar declination and \( W_s \) is the mean sunrise hour angle for the given month and \( n \) is the number of days of the year starting from 1\(^{st}\) of January to 31\(^{st}\) of December.
The solar declination, \( \delta \) and the mean sunrise hour angle, \( W_s \) can be calculated using the following equations [14] and [27]:

\[
\delta = 23.45 \sin \left( \frac{360(223+n)}{365} \right)
\]

(3)

\[
W_s = \cos^{-1}( -\tan \phi \tan \delta)
\]

(4)

For a given month, the maximum possible sunshine duration (monthly average day length \( (S_o) \) ) can be determined using [14] and [27] by

\[
S_o = \frac{2}{15} W_s
\]

(5)

The clearness index \( (K_r) \) is defined as the ratio of the observed/measured horizontal terrestrial solar radiation \( H \), to the calculated/predicted/estimated horizontal extraterrestrial solar radiation \( H_o \). The clearness index \( (K_r) \) gives the percentage deflection by the sky of the incoming global solar radiation and therefore indicates both level of availability of solar radiation and changes in atmospheric conditions in a given locality [11].

\[
K_T = \frac{H}{H_o}
\]

(6)

In this study, \( H_o \) and \( S_o \) were computed for each month using equations (2) and (5) respectively. The accuracy of the estimated values was tested by computing the Mean Bias Error (MBE), Root Mean Square Error (RMSE), and Mean Percentage Error (MPE), \( t \)-test and coefficient of correlation \( (R) \). The expressions for the MBE, RMSE and MPE as stated according to [10] are given as follows.

\[
MBE = \frac{1}{n} \sum_{i=1}^{n} (H_{i,calc} - H_{i,mea})
\]

(7)

\[
RMSE = \left( \frac{1}{n} \sum_{i=1}^{n} (H_{i,calc} - H_{i,mea})^2 \right)^{1/2}
\]

(8)

\[
MPE = \frac{1}{n} \left( \frac{H_{i,mea} - H_{i,calc}}{H_{i,mea}} \right) \times 100
\]

(9)

The \( t \)-test defined by student [8] in one of the tests for mean values, the random variable \( t \) with \( n-1 \) degrees of freedom may be written as follows.

\[
t = \left[ \frac{(n-1)(\text{MBE})^2}{(\text{RMSE})^2 - (\text{MBE})^2} \right]^{1/2}
\]

(10)

From equations (7), (8), and (9) above \( H_{i,mea}, H_{i,calc} \) and \( n \) are respectively the \( i \)\(^{th}\) measured and \( i \)\(^{th}\) calculated values of daily global solar radiation and the total number of observations.\([2, 9] \) and [13 - 14] have recommended that a zero value for MBE is ideal and a low RMSE is desirable. Furthermore, the smaller the value of the MBE, RMSE and MPE the better is the model’s performance. The RMSE test provides information on the short-term performance of the studied model as it allows a term – by – term comparison of the actual deviation between the calculated values and the measured values. The MPE test gives long term performance of the examined regression equations, a positive MPE and MBE values provide the averages amount of overestimation in the calculated values, while the negative values gives underestimation. For a better model performance, a low value of MPE is desirable and the percentage error between –10% and +10% is considered acceptable [19]. The smaller the value of \( t \) the better is the performance. To determine whether a model’s estimates are statistically significant, one simply has to determine, from standard statistical tables, the critical \( t \)-value, i.e., \( t_{\alpha/2} \) at \( \alpha \) level of significance and \( (n-1) \) degrees of freedom. For the model’s estimates to be judged statistically significant at the \( (1 - \alpha) \) confidence level, the computed \( t \) value must be less than the critical value. Similarly, for better data modelling, the coefficient of correlation \( R \) and coefficient of determination \( R^2 \) should approach 1 (100%) as closely as possible. The various equations used in this study for estimating the global solar radiation with the clearness index been the dependent variable and the sunshine duration as independent variables are shown on Table 1.

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Model Type</th>
<th>Regression equation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear</td>
<td>( \frac{H}{H_o} = a + b \left( \frac{S}{S_o} \right) )</td>
<td>[5] and [23]</td>
</tr>
<tr>
<td>2</td>
<td>Quadratic</td>
<td>( \frac{H}{H_o} = a + b \left( \frac{S}{S_o} \right) + c \left( \frac{S}{S_o} \right)^2 )</td>
<td>[21]</td>
</tr>
<tr>
<td>3</td>
<td>Cubic</td>
<td>( \frac{H}{H_o} = a + b \left( \frac{S}{S_o} \right) + c \left( \frac{S}{S_o} \right)^2 + d \left( \frac{S}{S_o} \right)^3 )</td>
<td>[26]</td>
</tr>
<tr>
<td>4</td>
<td>Linear logarithmic</td>
<td>( \frac{H}{H_o} = a + b \ln \left( \frac{S}{S_o} \right) + c \ln \left( \frac{S}{S_o} \right)^2 )</td>
<td>[20]</td>
</tr>
<tr>
<td>5</td>
<td>Logarithmic</td>
<td>( \frac{H}{H_o} = a + b \ln \left( \frac{S}{S_o} \right) )</td>
<td>[4]</td>
</tr>
<tr>
<td>6</td>
<td>Linear exponential</td>
<td>( \frac{H}{H_o} = a + b \left( \frac{S}{S_o} \right) + c \exp \left( \frac{S}{S_o} \right) )</td>
<td>[6]</td>
</tr>
<tr>
<td>7</td>
<td>Exponential</td>
<td>( \frac{H}{H_o} = a + b \exp \left( \frac{S}{S_o} \right) )</td>
<td>[1]</td>
</tr>
<tr>
<td>8</td>
<td>Linear</td>
<td>( \frac{H}{H_o} = a + b \left( \frac{S}{S_o} \right) )</td>
<td>[17]</td>
</tr>
<tr>
<td>9</td>
<td>Exponent</td>
<td>( \frac{H}{H_o} = a \left( \frac{S}{S_o} \right)^b )</td>
<td>[6]</td>
</tr>
<tr>
<td>10</td>
<td>Linear, latitude related</td>
<td>( \frac{H}{H_o} = 0.29 \cos (\phi) + 0.52 \left( \frac{S}{S_o} \right) )</td>
<td>[12]</td>
</tr>
<tr>
<td>11</td>
<td>Linear</td>
<td>( \frac{H}{H_o} = 0.23 + 0.48 \left( \frac{S}{S_o} \right) )</td>
<td>[22]</td>
</tr>
<tr>
<td>12</td>
<td>Linear</td>
<td>( \frac{H}{H_o} = 0.18 + 0.62 \left( \frac{S}{S_o} \right) )</td>
<td>[24]</td>
</tr>
</tbody>
</table>

Model 8 is a modification of the Angström-Prescott model through the use of the ratio of \( \left( \frac{S}{S_{nh}} \right) \) instead of \( \left( \frac{S}{S_o} \right) \) by [17] and \( \left( \frac{S}{S_{nh}} \right) \) is given by the relation

\[
\left( \frac{S}{S_{nh}} \right) = \frac{S}{S_o} \left( \frac{S_o}{S_{nh}} \right)
\]
The regression equations developed in this study based on the adopted models using Minitab 16 program software are as follows:

\[
\frac{H}{H_0} = 0.141 + 0.749 \left( \frac{S}{S_0} \right) \quad (R^2 = 68.4\%) \tag{12}
\]

\[
\frac{H}{H_0} = -0.244 + 1.97 \left( \frac{S}{S_0} \right) - 0.96 \left( \frac{S}{S_0} \right)^2 \quad (R^2 = 69.0\%) \tag{13}
\]

\[
\frac{H}{H_0} = -5.26 + 25.9 \left( \frac{S}{S_0} \right) - 38.6 \left( \frac{S}{S_0} \right)^2 + 19.6 \left( \frac{S}{S_0} \right)^3 \quad (R^2 = 70.6\%) \tag{14}
\]

\[
\frac{H}{H_0} = 1.42 - 0.63 \left( \frac{S}{S_0} \right) + 0.88 \ln \left( \frac{S}{S_0} \right) \quad (R^2 = 69.2\%) \tag{15}
\]

\[
\frac{H}{H_0} = 0.836 + 0.476 \ln \left( \frac{S}{S_0} \right) \quad (R^2 = 69.1\%) \tag{16}
\]

\[
\frac{H}{H_0} = 0.80 + 2.59 \left( \frac{S}{S_0} \right) - 0.97 \exp \left( \frac{S}{S_0} \right) \quad (R^2 = 69.0\%) \tag{17}
\]

\[
\frac{H}{H_0} = -0.126 + 0.392 \exp \left( \frac{S}{S_0} \right) \quad (R^2 = 67.8\%) \tag{18}
\]

\[
\frac{H}{H_0} = 0.140 + 0.857 \left( \frac{S}{S_{nh}} \right) \quad (R^2 = 68.4\%) \tag{19}
\]

\[
\frac{H}{H_0} = 0.884 \left( \frac{S}{S_0} \right)^{0.798} \quad (R^2 = 68.7\%) \tag{20}
\]

\[
\frac{H}{H_0} = 0.29 \cos(\phi) + 0.52 \left( \frac{S}{S_0} \right) \quad (R^2 = 68.4\%) \tag{21}
\]

\[
\frac{H}{H_0} = 0.23 + 0.48 \left( \frac{S}{S_0} \right) \quad (R^2 = 68.4\%) \tag{22}
\]

\[
\frac{H}{H_0} = 0.18 + 0.62 \left( \frac{S}{S_0} \right) \quad (R^2 = 68.4\%) \tag{23}
\]

It is interesting to note here that equations (21 – 23) have the same values as those of models (10 – 12) as the empirical constant “a” and “b” have already been determined.

3. RESULTS AND DISCUSSION

Table 2: Input parameters for estimation of monthly average daily global solar radiation for Zaria (1980 – 2010)

<table>
<thead>
<tr>
<th>Month</th>
<th>$\Delta H/H_0$</th>
<th>$S/S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.6823</td>
<td>0.7185</td>
</tr>
<tr>
<td>Feb</td>
<td>0.6875</td>
<td>0.6867</td>
</tr>
<tr>
<td>Mar</td>
<td>0.6666</td>
<td>0.5604</td>
</tr>
<tr>
<td>Apr</td>
<td>0.6338</td>
<td>0.5983</td>
</tr>
<tr>
<td>May</td>
<td>0.5914</td>
<td>0.6480</td>
</tr>
<tr>
<td>Jun</td>
<td>0.5535</td>
<td>0.6109</td>
</tr>
<tr>
<td>Jul</td>
<td>0.5055</td>
<td>0.5142</td>
</tr>
<tr>
<td>Aug</td>
<td>0.4819</td>
<td>0.5188</td>
</tr>
<tr>
<td>Sep</td>
<td>0.5687</td>
<td>0.6009</td>
</tr>
<tr>
<td>Oct</td>
<td>0.6522</td>
<td>0.7029</td>
</tr>
<tr>
<td>Nov</td>
<td>0.7124</td>
<td>0.7781</td>
</tr>
<tr>
<td>Dec</td>
<td>0.7004</td>
<td>0.7359</td>
</tr>
</tbody>
</table>

The statistical test of R, MBE, RMSE, MPE and t – test were determined during the period of thirty one years for Zaria-Nigeria. The results are displayed in Table 3. According to the statistical test results, it can be seen that the estimated values of the monthly average daily global solar radiation are in favorable agreement with the measured values for all the models except model 11 and to some extend model 12. According to the statistical test result of coefficient of correlation (R), all the models achieved a good result (R ≥ 82.70%) for the study area. This implies that the models obtained showed some reasonably level of compatibility with the measured data. However, model 3 has the highest value. According to the statistical test of MBE all the models exhibited underestimation in the estimated value except for model 1, 3 and 7 which exhibited overestimation in the estimated value. Model 5 has the lowest value of MBE (–0.0045 $Mjm^{-2}day^{-1}$) and was judged the best performing model while model 11 has the highest value and was judged the weakest. According to the statistical test result of RMSE, model 4 has the lowest value of RMSE (1.5290 $Mjm^{-2}day^{-1}$) and was judged the best performing model while model 11 has the highest value and was judged the weakest. According to the statistical test result of MPE, it was found that the models (1 – 10) and model 12 shows a good result as they are in the range of acceptable values between –0.0012% and +6.3912% while model 11 ceeds±10%. However, models 2 has the lowest value of MPE (–0.0012%) and was judged the best performing model. According to the statistical test result of t – test ($t_{critical} = 2.20$) at 95% confidence level and ($t_{critical} = 3.12$) at 99% confidence level. The comparison between the different models according to t – value shows that the estimated values for models (1 – 10) were less than the critical t – value indicating statistical significance at 95% and 99% confidence level while model 11 and 12 are not significant at 95% and 99% confidence level. The t – test shows that model 5 is the best performing model with the lowest value, t – value(0.0099). It is important to note here that R and MPE are measured in % while MBE and RMSE are in $Mjm^{-2}day^{-1}$.
suggested values using R, Aug illustrated in Figure 1. As can be seen from the figure that a reasonably agreement between the measured and estimated values was obtained for the models (1 – 10). It was obvious from the figure that a considerable deviation occurred between the measured and estimated values using model 11 and 12 showing underestimation in the estimated values for all the months of the year when compared to the measured and other models (1 – 10). However, a noticeable underestimation for all the models was observed in the months from February – April and overestimation in the estimated values of model 3 in the months of May, June and from September – December.

4 CONCLUSION
Solar energy technologies offer opportunities for use of a clean, renewable and domestic energy resource and are essential component for the future sustainable energy. Availability of a complete and accurate solar radiation data base for each specific location is paramount in the study, prediction and design of solar energy systems. Information on global solar radiation at any location over a long period of time is useful not only to the locality where the radiation data is collected but also for the world at large. In this present study, twelve (12) empirical sunshine based models have been selected from literatures to estimate the global solar radiation for Zaria-Nigeria during the period of thirty one years. The selected models were statistically tested using the performance indicators of R, MBE, RMSE, MPE and the t – test. The regression equations (16, 15 and 13) which are the modified forms of models 5, 4 and 2 are extremely recommended based on their outstanding performances for estimating global solar radiation in Zaria-Nigeria. On the other hand, models 11 and 12 are not recommended to be used to estimate global solar radiation in Zaria-Nigeria. This present study is believed to advance the state of knowledge of global solar radiation to the point where it has applications in the prediction of daily global solar radiation, not only in Zaria also in regions of similar climatic information.

ACKNOWLEDGMENT
The authors are grateful to the management and staff of the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos for providing all the necessary data used in this study.

REFERENCES

Figure 1: Comparison between the measured and estimated global solar radiation for Zaria

![Figure 1: Comparison between the measured and estimated global solar radiation for Zaria](image-url)
tion between solar radiation and hours of sunshine, Journal of Royal Meteorological Society., 84, 172 – 175.


