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An Analytical Study Of Convection In A Mushy Layer Under Gravity Inclination

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Abstract: The problem of weekly nonlinear convection in a horizontal mushy layer under suitable assumptions and approximations in a gravity inclined environment is studied analytically by applying a modified perturbation technique. The finite amplitude solution for the weakly nonlinear problem is obtained under a near eutectic approximation, large far field temperature and for large Stefan number in a gravity inclined environment. Specific information with regard to the convective solutions are obtained with regard to the behaviour of the system. From the results it appears that the chimney formation gets promoted as the system gets destabilised when the concentration ratio is increased while, the formation of chimneys occurs at a larger Darcy Rayleigh number when the concentration ratio is decreased. Also, it is found that the gravity inclination reduces the tendency for chimney formation under the given physical conditions. It is concluded that by a proper choice of the inclination parameter and the governing parameters, it is possible to have a good control over the formation of chimneys convection so that the appearance of freckles in the resulting solid during the solidification process could be avoided. The results are in excellent agreement with the available results in the limiting cases.

Keywords: Mushy layer; Weakly nonlinear; Analytical approach; Chimney convection; Gravity inclination.

1. INTRODUCTION

A region called “mushy layer” that comprises of partially solidified reactive porous materials like, solid crystals and interstitial melt, will be formed during the solidification of a binary alloy due to the morphological instability of the solid-liquid interface [1]. Thus a mushy layer is a reactive two-phase system where the solid matrix and the residual liquid exist. The understanding of the complex interactions between the flow of melt and the solidification is interesting and is of great importance since the transfer of heat and mass associated with the fluid flow has a profound influence on the process of solidification and becomes responsible for the chimney formation which causes imperfections in the resulting solid. The interesting phenomenon that occurs within the solidifying melt is the formation of chimneys which are narrow dendrite-free cylindrical regions of zero solid fraction and are very much similar to the imperfections called ‘freckles’ that appear in the casting of metallic alloys [2],[3]. The formation of chimneys during the solidification of a binary or a multicomponent alloy constitutes three stages viz., finger, plume and chimney convections [4],[5]. Actually, during the solidification process the solidification front or the interface between the solid and the liquid becomes highly dendritic due to the morphological instability. As a consequence there will be a formation of a region called ‘mushy layer’ consisting of a partially solidified melt, the dendritic structure of which is quite complex [1]. Then the system becomes unstable due to the density gradient that results from the rejected materials and there will be a transition to convection. In other words, the transition to convection in the mushy layer drives the fluid flow into the regions with differential densities, which in turn becomes responsible for the solidification or dissolution of the solid matrix and eventually affects the material properties. Further, as a result of the interaction of the thermal fields and the generated convective motions, chimneys which are responsible for the imperfections in the resulting solid will be formed [2],[6],[7],[8],[9].The review article by [10] discusses the striking fluid-mechanical events that take place during the solidification process. It is of importance and interest to study the resulting buoyancy driven convection in a mushy layer. A good knowledge about the onset of convection, dynamics of the mushy layer and the chimney convection is absolutely essential since they have significant applications in areas viz, geophysics for predicting brine fluxes from sea ice [11],[12], geology, industries Sea dynamics[13], metallurgy because of the heterogeneous nature of the material properties associated with the chimneys which lead to imperfections in metal castings[14] and also in the solidification process within the earths interior[15] Especially in metallurgy and dynamics of sea and geophysics, the mechanism and the process of formation of chimneys which spoil the quality, physical properties and the internal structure of the resulting solid, are important study areas [16],[17],[18]. In the past three decades the study pertaining to the development of different convective models and analysis for the case of convection in mushy layers has attracted researchers [18],[8]. The works connected with the formulation of the governing equations in the study of convection in mush layers, the development of mathematical models and also the solution procedure are available [19],[20]. The mathematical models describing the characteristics of mushy layer and the related phenomena are exclusively based on the key feature that the length scale of the internal boundaries are extremely small when compared to the macroscopic dimensions of the mushy layer. Linear and weakly nonlinear convective instabilities in a mushy layer have been studied by quite a number of researchers under different types of assumptions and approximations [21], [6], [22], [23], [24], [25], [27]. Quite a number of works on convective flow in a mushy layer is available. A detailed review on convection in mushy layers is given by [6], [27]. Recently, [28], [29], [30] have applied weakly nonlinear evolution approach to study two-dimensional convective motions in a mushy layer with

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impermeable solidification front under different situations. Finally, [31] have studied numerically the effects of inertia on connection in a mushy layer with constant permeability. Analytically, [32], [33] have studied the effects of inertia on convection in a passive mushy layer. The main objective behind these studies is to study the history of the solidification process that could yield pure solid and facilitate the suppression of the freckles which have catastrophic effects on the internal structure of the resulting solid. A thorough survey of the literature pertaining to the subject reveals that no in depth analytical work is available for the case of convection in a mushy layer with and without constraints. Therefore, the present analytical work is carried out to study the effects of large Stefan number, Concentration ratio, gravity inclination and the permeability function on the total Rayleigh number, velocity and local solid fraction profiles in a mushy layer near eutectic temperature. The boundaries are assumed to be impervious so that the Darcy's equation is valid. The paper is organised as follows: Section 1, deals with a brief introduction to the study together with the literature survey. Section 2, discusses the mathematical formulation. In sections 3 and 4, the basic state, the linear stability and weakly nonlinear analyses are presented. Finally, section 5, presents the results and discussions of the study.

2. MATHEMATICSL FORMULATION

The model consists of a horizontal mushy layer formed during the solidification of a binary alloy as shown in figure 1. The process of uniform cooling from below of the system results in the upward advancement of the solid – mush and mush – liquid interface with a constant solidification speed V_0. In other words the mushy layer is sandwiched between the solid and the liquid regions. The study is carried out in a moving frame of reference,

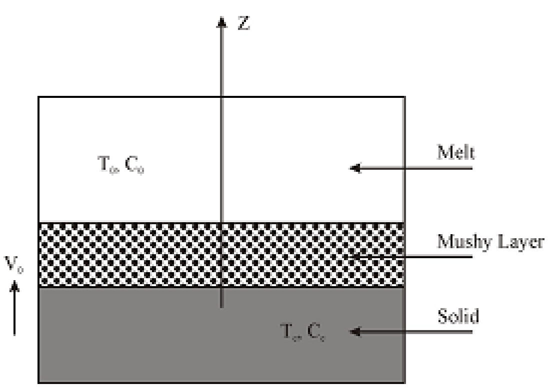

***Fig.1**. The schematic diagram of the physical system*

where the bottom boundary of the mushy layer z = 0 is kept at the eutectic temperature $T = T_E$, while the top boundary z = d is kept at the liquidus temperature T_L (C_0) and the mixture of the composition C_0 is supplied through the surface. The system is also subject to a gravity inclination where the vector g = (Sin α^* , -Sin α^* , Cos α^*) and α^* is the angle of inclination of the gravity vector to the vertical axis.Following are the assumptions made for the study:

i. The top and the bottom boundaries of the mushy layer are assumed to be isothermal non-deformable and impermeable to the fluid flow, so that the mushy layer is kept dynamically isolated from the other components of the system [23].

ii. The solidification front (the frame of reference) is moving upwards with a velocity V_0 relative to the solid formed and the solid dendrites within the mushy layer. This makes the basic state to be steady.

iii. The temperature T and the composition C of the liquid in the mushy layer are required to satisfy a linear liquidus relationship $T = T_L(C) = T_L(C_0) + \Gamma (C - C_0)$, where Γ is a constant. The liquid is assumed to be Newtonian with a linearized equation of state $\rho = \rho_0$ [1 + β(C- C_0)] where ρ is the density of the liquid, ρ_0 is a reference density, $\beta = \beta^* - \alpha^*\Gamma$, α^* and β^* are constant expansion coefficients for heat and solute respectively.

iv. First following [23], we study a limit in which the thickness of the mushy layer is much less than the diffusion length scale by letting $\delta \leq 1$.

v. Secondly we assume that the compositional ratio is large by writing $C_R = \frac{C_R}{\delta}$ with $C_R = 0(1)$ as $\delta \rightarrow 0$ which corresponds to the near eutectic approximation introduced by [34].

vi. we consider the limit in which the Stefan number is large[29] by taking S = S/ δ with S = O(1) as $\delta \rightarrow 0$ which corresponds to the situation in which the latent heat liberated during the local phase change is much larger than the heat associated with the typical variations of temperature across the mushy layer. Note that the particular scaling allows the destabilisation of the system to an oscillatory mode of convection [25],[35]. However, that a key implication of the near-eutectic approximation($C = O(\delta^{-1})$) is that the solid fraction is small, and hence the permeability is uniform to the lowest order. As a consequence, we follow [23] and expand the permeability in terms of the small solid fraction Φ,

$K(\Phi)=1+ K_1\Phi + K_2\Phi^2$ + ----- (1)

vii. where, on physical grounds, we demand that K_1, K_2, etc. are non-negative. Under the above assumptions and approximations the governing equations of the systems are Conservation of momentum, Conservation of mass, Conservation of heat and solute. These equations in the dimensionless form by using the scales viz., $\frac{V_0}{\pi_0}$, ΔT, $\frac{\kappa}{V_0^2}$, $\frac{\kappa\mu}{\pi_0}$, $\frac{\kappa}{V_0}$ for the variables velocity, temperature, time, pressure and length respectively, are :

$$K(\Phi)u= -\frac{\partial p}{\partial x} - R\Theta \mathrm{Sin}\alpha^* \quad (2a)$$

$$K(\Phi)v= -\frac{\partial p}{\partial y} + R\Theta \mathrm{Sin}\alpha^* \quad (2b)$$

$$K(\Phi)w= -\frac{\partial p}{\partial z} - R\Theta \mathrm{Cos}\alpha^* \quad (2c)$$

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$\nabla.\vec{q}=0.$ (3)

$(\partial_t-\partial_z)(\theta-S\Phi)+\vec{q}\,\nabla\theta=\nabla^2\theta,$ (4)

$(\partial_t-\partial_z)((1-\Phi)\theta+C\Phi)+\vec{q}\,\nabla\theta=0,$ (5)

Next we eliminate the pressure in the above set of equations by applying the following transformation:
To start with, the application of the transformation

$\frac{\partial}{\partial Z}[\frac{\partial(2a)}{\partial x}+\frac{\partial(2b)}{\partial y}]-\nabla_1^2(2c)$ (6)

yields

$\nabla^2(Kw)-\frac{\partial}{\partial z}(u.\Delta k)=R.\,Sin\alpha^*\frac{\partial}{\partial z}\frac{\partial}{\partial x}\Theta$

$-R.\,Sin\,\alpha^*\frac{\partial}{\partial z}\frac{\partial}{\partial y}\Theta-R.\nabla_1^2\,Cos\alpha^*$ (6a)

Next by applying the transformation, we get

$\frac{\partial}{\partial x}[\frac{\partial(2b)}{\partial y}+\frac{\partial(2c)}{\partial z}]-\nabla_1^2(2a)$ (7)

$\nabla^2(Ku)-\frac{\partial}{\partial x}(u.\Delta K)=-R.\,Sin\alpha^*\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Theta$

$+R.\,Cos\,\alpha^*\frac{\partial}{\partial x}\frac{\partial}{\partial z}\Theta-R.\,\nabla_1^2\,Sin\alpha^*$ (7a)

Finally by applying the transformation,

$\frac{\partial}{\partial y}[\frac{\partial(2c)}{\partial z}+\frac{\partial(2a)}{\partial x}]-\nabla_1^2(2b)$ (8)

we get,

$\nabla^2(Kv)-\partial\frac{\partial}{\partial x}(u.\Delta K)=R.\,Cos\boldsymbol{\alpha}^*\frac{\partial}{\partial y}\frac{\partial}{\partial z}\Theta$

$+R.\,Sin\boldsymbol{\alpha}^*\frac{\partial}{\partial y}\frac{\partial}{\partial x}\Theta+R.\,\nabla_1^2\,Sin\boldsymbol{\alpha}^*\,\Theta$ (8a)

The associated boundary conditions are
$\boldsymbol{\Theta}=-1, w=0$ @z = 0,
$\boldsymbol{\theta}=0, w=0, \Phi=0$ @ z = δ. (9)
The above boundary conditions correspond to impermeable rigid boundaries of the mushy layer. The lower plate, between the solid and the mush, is maintained at the eutectic temperature T_E , while the upper boundary between the liquid and the mush (that is, at zero solid fraction Φ), is maintained at the far-field liquidus temperature $T_L(C_0)$.The porous medium is such that Darcy's law holds good.

Dimensionless parameters are the key parameters in the study through which the qualitative as well as quantitative behaviour of the system could be effectively studied. The function K(Φ) appearing in equation (2) measures the variation of permeability with respect to some zero-solid-fraction permeability π(0), assumed to be finite, such that

$K(\Phi)=\frac{\pi(0)}{\pi(\Phi)}$ (10)

The dimensionless parameters appearing in (2-5) are the Stefan number

$S=\frac{L}{C(T_L(C_0)-T_E)}$

the concentration ratio $\mathbf{C}=\frac{C_S-C_0}{C_0-C_E}$

the Rayleigh number $\mathbf{R}=\frac{\beta_{\Delta C.}\pi(0)g}{\mu V_0}$ (Rayleigh number)

and α^* is the inclination parameter (11)

where

$\Phi=1-l$: Local solid fraction, l – Local liquid fraction , P – Dynamic pressure, $\pi=\pi(\Phi)$: the permeability is a function of the local solid fraction , μ : Dynamic viscosity, t, T, k, l_h,γ are time, temperature, thermal diffusivity, specific heat, latent heat/unit mass, Cs: Composition of the solid phase ,C_0: Composition of the liquid phase ρ, ρ_0 : densities , $\vec{g}$ = (Sin α^* , -Sin α^* , Cos α^*) acceleration due to gravity, $\pi(0)$: The reference permeability, $\vec{q}$ = u i + vj +w k , , $\vec{q}$ is the Darcy velocity vector and (u,v,w) are the horizontal and vertical components of ,$\vec{q}$, i, j ,k : unit vectors along the x, y and z axes .δ : the dimensionless thickness, β: Expansion coefficient, T_∞: Far-field temperature, C_E : Eutectic composition. A sixth dimensionless parameter, appearing in the study is the dimensionless mushy layer thickness $\delta=d/(kV_0)$, appears in the boundary conditions. Before analysing the linear stability of the system the basic state analysis is carried out as follows:

3. BASIC STATE ANALYSIS

The steady motionless basic state system is considered here where each of the corresponding dependent variables is designated by a subscript "B". The basic state variables are assumed to be functions of z only.

$\Theta=\theta_B(z)+\varepsilon\,\hat{\theta}(x,y,z,t)$
$\Phi=\Phi_B(z)+\varepsilon\,\hat{\Phi}(x,y,z,t)$
$\vec{q}=0+\varepsilon\,\hat{q}(x,y,z,t)$
$P=P_B(Z)+\varepsilon\,P(x,y,z,t)$
$K=K_b(\Phi_B)+\varepsilon K(\Phi)$ (12)

where ε is the perturbation parameter(ε << 1) and the perturbed quantities can vary with respect to spatial and time variables. Using (12) in (13) we rescale the variables as

$(\partial_t-\delta\partial_z)(\theta-\frac{S}{\delta}\Phi)+\delta(\vec{q}\cdot\nabla)\theta=\nabla^2\theta,$ (13)

$(\partial_t-\delta\partial_z)((1-\Phi)\theta+\frac{C}{\delta}\Phi)+\delta(\vec{q}\cdot\nabla)\theta=0$ (14)

$K(\Phi)u=-\frac{\partial p}{\partial x}-R\Theta Sin\alpha^*$ (15a)

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$K(\Phi)v = -\frac{\partial p}{\partial y} + R\Theta Sin\alpha^*$ (15b)

$K(\Phi)w = -\frac{\partial p}{\partial z} - R\Theta Cos\alpha^*$ (15c)

$\nabla\cdot\vec{q} = 0.$ (16)

Equations (13 -16) allow a motionless steady basic state solution, depending only on the vertical position. By setting $\vec{q}=0$ and $\partial_t = 0$, we get

$-\delta D\theta_B + \vec{S}\, D\Phi_B = \nabla^2\theta_B,$ (17)

$-\delta D\theta_B + \delta D(\theta_B.\Phi_B) - \bar{C} D\theta_B = 0$ (18)

$0 = -D P_B + R\,\Theta_B\, Cos\alpha^*$ (19)

From (17-19), we have the following equations:

$\delta\, D\,\theta_B\,(\Phi_B - 1) + \delta\, D\Phi_B\,(\theta_B - \frac{C}{\delta}) = 0$

$\delta(1-\Phi_B)D\theta_B + D\Phi_B(\bar{C} - \delta\theta_B) = 0$ (20)

$D^2\theta_B + \delta D\theta_B - SD\Phi_B$ (21)

$DP_B - R\,\theta_B\, Cos\alpha^* = 0$ (22)

Boundary conditions are

$\theta_B = -1$ @ $z = 0$,

$\theta_B=0, \Phi_B=0$ @ $z=1$ (23)

On multiplying (22) by ($\bar{C} - \delta\,\theta_B$) and (21) by $\bar{S}$ *and adding , we get*

$(\bar{C} - \delta\,\theta_B)\,(D^2\theta_B + \delta\, D\,\theta_B - \bar{S}\, D\Phi_B) + \bar{S}\,(\bar{C}\,(1-\Phi_B)D\theta_B + (\bar{C} - \delta\theta_B)D\Phi_B) = 0$ (24)

Equations (21) - (24) are solved asymptotically by applying the following expansions

$\theta_B = \theta_{B0} + \delta\theta_{B1}$+------------------ $\Phi_B = \delta\,\Phi_{B0} + \delta^2\Phi_{B1}$+---------------- (25)

On substituting (25) in (24) and on equating the like powers of δ^0, δ and δ^2 we get, corresponding basic state system(17) to (19) and the solutions are given by

$\theta_{B0} = z-1,$ (26)

$\Theta_{B1} = -\frac{\Omega}{2}(z^2 - z),$ (27)

$\theta_{B2} = \frac{2(\Omega-1)}{C}[\frac{z^3}{6} - \frac{z^2}{2}] + \frac{\Omega^2}{2}[\frac{z^3}{3} - \frac{z^2}{2}] + [1 - \frac{2(\Omega-1)}{3C} + \frac{\Omega^2}{12}]z - 1$

(28)

$\Phi_{B0} = \frac{-1}{\bar{C}}[z-1]$ (29)

$\Phi_{B1} = -\frac{(z-1)^2}{\bar{C}^2} + \frac{\Omega}{2C}(z-1)^2\bar{C}$ (30)

Finally by substituting (26) to (30) in (25), we get

$\theta_B = (z-1) - \delta\frac{\Omega}{2}(z^2 - z) + \delta^2 [\frac{2(\Omega-1)}{C}[\frac{z^3}{6} - \frac{z^2}{2}] + \frac{\Omega^2}{2}[\frac{z^3}{3} - \frac{z^2}{2}] + [1 - \frac{2(\Omega-1)}{3C} + \frac{\Omega^2}{12}]z - 1]$+- (31)

$\Phi_B = -\delta\frac{1}{\bar{C}}[z-1] + \delta^2[-\frac{(z-1)^2}{\bar{C}^2} + \frac{\Omega}{2C}(z-1)^2]$ +---------

(32)

4. WEAKLY NONLINEAR ANALYSIS

In this section we first perform the linear stability analysis of the system to find the critical conditions and the growth rate σ and next consider a finite – amplitude perturbation expansion of the variables in order to predict the nonlinear effects on the system. For this purpose, we consider the expansions of the perturbation quantities in the form

$\theta = (\theta_{00} + \delta\theta_{01}$ + ---------$) + \varepsilon(\theta_{10} + \delta\theta_{11}$ + -----------$) + \varepsilon^2(\theta_{20} + \delta\theta_{21}$ + ----- ---)

$\Omega\Phi = (\Phi_{00} + \delta\,\Phi_{01}$ + -----$) + \varepsilon(\Phi_{10} + \delta\,\Phi_{11}$ + -----$) + \varepsilon^2(\frac{1}{\delta}\Phi_{2(-1)} + \delta\,\Phi_{20}$ + --------)

$\vec{q}\,\Omega^{\frac{1}{2}} = (\vec{q}_{00} + \delta\,\vec{q}_{01}$ + ---------$) + \varepsilon(\vec{q}_{10} + \delta\,\vec{q}_{11}$+ ---------$) + \varepsilon^2(\vec{q}_{20} + \delta\,\vec{q}_{21}$ + -----)

$\Omega^{1/2}R = (R_{00} + \delta\, R_{01}$ +--------$) + \varepsilon(R_{10} + \delta\, R_{11}$+--------$) + \varepsilon^2((R_{20} + \delta R_{21}$+----) (33)

From (33) it is observed that the equation is singular at the order of ϵ^2 as $\delta > 0$. In fact this is the contribution of the forcing term Φ_{00} from the equation for Φ at the order of ϵ^2.

Therefore the balance is made by adding the term $(1/\delta)\,\Phi_{2(-1)}$ at order of ϵ^2.

Further, the asymptotic expansion of (33) is meaningful only when $\epsilon^2\delta$ is very much less than 1. In order to study the stability characteristics of the problem, we substitute (33) into (3), (4), (6a) ,(7a) and (8a) and collecting the terms of order δ^{-1} and δ^0, we get the following set of differential equations:

$\sigma_{00} = 0$ (34)

$S(\sigma_{01} - D)\Phi_{00} - R_{00}w_{00} + \nabla^2\theta_{00} = 0$ (35)

$(\sigma_{01} - D)C\Phi_{00} + R_{00}W_{00} = 0$ (36)

$\nabla^2 w_{00} = ikR_{00}Sin\alpha^* D\theta_{00} - R_{00}Cos\alpha^*\nabla_1^2\theta_{00}$ (37)

$\nabla^2 u_{00} = ikR_{00}Cos\alpha^* D\theta_{00} - \nabla_1^2 R_{00}Sin\alpha^*\theta_{00}$ (38)

The set of differential equations (34) to(38) is solved along with the boundary conditions by letting

$\Theta_{00} = -f_k Sin(\pi z)$ (39)

$w_{00} = \frac{(\pi^2 + \alpha^2)Sin(\pi z)}{\Omega\, R_{00}}$ (40)

$$\frac{R_{00}.Sin\alpha^*.K^2\,\theta_{00} - \pi.R_{00}.Cos\alpha^*\,(ik)f_k.Cos\,\pi z + k^2 R_{00}.Sin\alpha^*.\theta_{00}}{-(\pi^2 + K^2)}$$

(41)

$$R_{00}=\frac{(\pi^2+\alpha^2)}{\alpha\sqrt{\Omega}} \tag{42}$$

$$\Phi_{00}=\frac{-\pi f k(\pi^2+\alpha^2)}{\Omega\, C\,(\pi^2+\frac{\sigma^2}{\delta^2})}\left[e^{\frac{\alpha}{\delta}(Z-1)}+\frac{\sigma}{\delta\pi}Sin(\pi z)+Cos(\pi z)\right] \tag{43}$$

Next we consider the system of o(δ) :

$$\nabla^2\Theta_{01} - R_{00}\,(w_{01} + w_{00}D\Theta_{01}) - R_{01}\,w_{00} + D\Theta_{00} - S.D\Theta_{01} = 0 \tag{44}$$

$$D\Phi_{01}-\frac{R_{00}}{C}w_{01}-\frac{1}{C}(R_{00}w_{00}D\Theta_{01}+R_{01}w_{00}) +\frac{1}{C}D\Theta_{00}-D\Theta_{00}.\Phi_{00}=0 \tag{45}$$

$$\nabla^2 w_{01} + k_1.\nabla^2(\Phi_{B0}.W_{00}) = R_{00}.\,Sin\alpha^*.\frac{\partial}{\partial z}\frac{\partial}{\partial x}\Theta_{01} + R_{01}.\,Sin\alpha^*.\frac{\partial}{\partial z}\frac{\partial}{\partial x}\Theta_{00} - R_{00}.\,Sin\alpha.\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Theta_{01} - R_{01}.\,Sin\alpha^*.\frac{\partial}{\partial z}\frac{\partial}{\partial y}\Theta_{00} + R_{00}.k^2Cos\alpha^*.\,\Theta_{01} + R_{01}.k^2\,Cos\alpha^*.\,\Theta_{00} + K_1D\,(D(\Phi_{B0}.W_{00})) \tag{46}$$

$$\nabla^2(u_{01} + k1.\,\Phi_{B0}.u_{00}) - \frac{\partial}{\partial x}[W_{00}.\frac{\partial}{\partial z}k1.\,\Phi_{B0}] = -R_{00}.\,Sin\alpha^*.\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Theta_{01} - R_{01}.\,Sin\alpha^*\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Theta_{00} + R_{00}.\,Cos\alpha^*.\frac{\partial}{\partial x}\frac{\partial}{\partial z}\Theta_{01} + R_{01}\,Cos\alpha^*\frac{\partial}{\partial x}\frac{\partial}{\partial z}\Theta_{00} - \nabla_1^2.R_{00}.\,Sin\alpha^*\,\Theta_{01} - \nabla^2 R_{01}.\,Sin\alpha^*\,\Theta_{00} \tag{47}$$

In (47) to (49), we eliminate W_{01} and Φ_{01} so that

$$(D^2-\alpha^2)^2\,\Theta_{01} + R_{00}\,\Omega\,\{K_1(D^2-\alpha^2)(\Phi_{B0}W_{00}) - K_1D(D\Phi_{B0}W_{00}) - Cos\alpha^*\alpha^2(R_{00}\Theta_{01}+R_{01}\Theta_{00}) - ik\,Sin\,\alpha^*(R_{00}D\Theta_{01}+R_{01}D\Theta_{00}) - (D^2-\alpha^2)\,D(\Theta_{B1}\Phi_{00})\} - \Omega\,\{R_{01}(D^2-\alpha^2)\,w_{00} + (D^2-\alpha^2)\,D\Theta_{00}\} - \frac{S}{C}(D^2-\alpha^2)\,D(\Theta_{B0}\Phi_{00}) = 0 \tag{48}$$

Multiplying the above equation by Θ_{00} and applying the solvability and orthogonality property, we obtain:

$$R_{00}^2=\frac{(\pi^2+\alpha^2)^2}{\alpha\Omega Cos\alpha^*} \tag{49}$$

where

$$\Omega = 1+\frac{S}{C}$$

On Substituting for R_{00} and simplifying we get,

$$R_{00}=\left(\frac{(\pi^2+\alpha^2)}{\alpha\sqrt{\Omega Cos\alpha^*}}\right) \tag{50}$$

$$R_{01}=\frac{1}{4}\left[\frac{k1}{c}-\frac{s\pi}{(\pi^2+\alpha^2)}\right]R_{00}$$

$$\Rightarrow R_{01}=\frac{1}{4}\left[\frac{k1}{c}-\frac{s\pi}{(\pi^2+\alpha^2)}\right]\left(\frac{(\pi^2+\alpha^2)}{\alpha\sqrt{\Omega Cos\alpha^*}}\right) \tag{51}$$

Hence ,

$$R_{01}=\frac{1}{4\sqrt{\Omega Cos\alpha^*}}\left[\frac{K_1(\pi^2+\alpha^2)}{c}-S\,\pi\right] \tag{52}$$

Further, from $R = R_{00} + \delta\, R_{01}$ we can write

$$R_C=\frac{(\pi^2+\alpha^2)}{\alpha\sqrt{\Omega.Cos\alpha^*}}\left[1+\frac{\delta}{4}\left(\frac{k1}{c}-\frac{s\pi}{(\pi^2+\alpha^2)}\right)\right] \tag{53a}$$

and for $\alpha_c = \pi$

$$R_C=\frac{2\pi}{\sqrt{\Omega.Cos\alpha^*}}\left[1+\frac{\delta}{4}\left(\frac{k1}{c}-\frac{s}{\pi}\right)\right] \tag{53b}$$

5. RESULTS AND DISCUSSION

Marginal stability curves are presented in figures 2-4, where the graphs of total R vs α for the experimental values [35], δ = 0.1, 0.3, 0.6; S = 3.2, C = 9 , K_1 = 0.2, Ω = 1.35556 are plotted respectively.

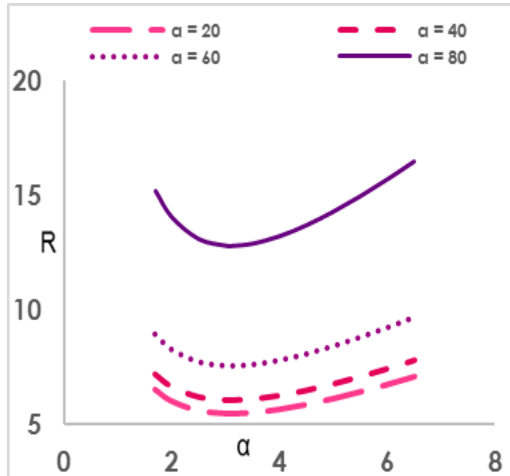

Fig. 2. *Total R For δ = 0.1*

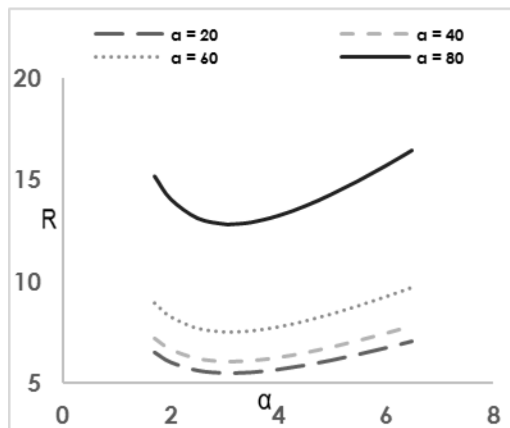

Fig. 3. *Total R For δ = 0.3 K ≠ π*

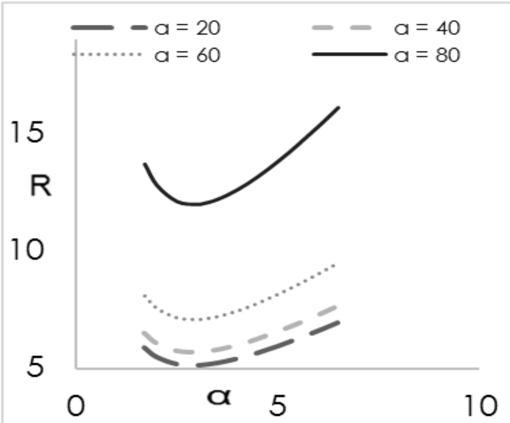

Fig. 4. *Total R For δ = 0.6 K ≠ π*

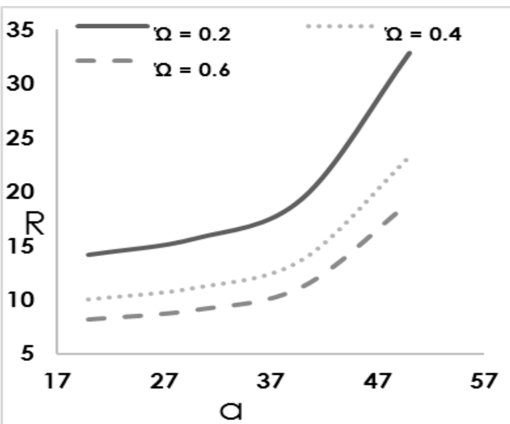

Fig. 5. *Total R for K = π, S < C*

The following observations are made: (i) The critical wave number decreases as δ increases i.e., α_c = 3.1416 for δ = 0.1 and α_c = 3 for δ = 0.3 and 0.6 (ii) The effect of inclination on convection in a mushy layer is that for a particular wave number there is a drastic increase in the value of R (iii) but, the value of R decreases as δ increases. In other words the enhancement in the values of R for large inclination suggests that inclination facilitates the non-formation of freckles in the resulting crystal during the solidification process. In fig. 5, the graph of R vs α* is presented for Ω = 0.2, 0.4, 0.6 and S/C < Ω respectively. The figure suggests that for a particular angle of inclination, the increase in the value of Ω from 0.2 to 0.6 drastically reduces the value of R. But, for a particular value of Ω the increase in α* from 20 to 60, drastically increases the value of R. Thus, it is observed that the system gets more stabilized as α* increases irrespective of the value of Ω. Further it is found that there is a slight enhancement in the values of R for S/C > Ω when compared to those of S/C < Ω.

CONCLUSION:

The present analytical study concentrates in predicting the effect of gravity inclination on convection in a mushy layer formed during the solidification process of a binary or multicomponent alloy. The study is based on a near eutectic temperature approximation and large far-field temperature. A modified perturbation technique is employed and the solutions corresponding to the basic, first and second order systems are determined. Marginal stability curves are presented for the experimental values [36] of Stefan number, Concentration ratio and the inclination parameter. Some of the important results are : (i) The mushy layer thickness has its significant influence in predicting the critical wave number with regard to total R (ii) There is an increase in the value of total R for a particular value of α and Ω in both the cases (S/C < Ω and S/C > Ω). But Ω has a destabilizing effect on the system, therefore it is concluded that by the proper choice of the angle of inclination the chimney convection could be inhibited to a great extent by employing the present analytical technique so that the resulting solid would be free from freckles. Our results are in excellent agreement with those of [33],[37]

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