Non-Newtonian Blood Flow In A Stenosed Artery With Porous Walls In The Present Of Magnetic Field Effect

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ABSTRACT: This paper is introduced the impact of magnetic field on a Non-Newtonian biomagnetic fluid. The stenosis artery with porous media model is used for solving the magnetic field and fluid equations simultaneously. The lump in stenosis artery creates vortexes after it place which disrupt the vessel flow. The external magnetic field is applied to weaken this vortexes and produce uniform flow. Furthermore the magnetic field decreases the shear stress on the artery due to neutralizing the created vortex. Another significant effect of applying magnetic force is declining the pressure in this region such that ten times higher magnetic field intensity from $10^{11}$ results 8.6X and 6.5X lower maximum pressure and shear stress on artery walls respectively.

Keywords : Biomagnetic, Heat Transfer, Porosity, Stenosis Artery

1 INTRODUCTION

Nowadays, the thickness of intimal is the main reason of cardiovascular diseases and formation of atherosclerosis. Insofar as stenosis artery disease take the life of many people. In atherosclerotic, arteries are generally narrowed and the inner walls are covered by oil material. The main reason of stenosis formation is backlogging of macromolecules on arterial walls. This lump clogs the blood flow and can cause infarct. There are numerous ways for controlling the blood flow [1]. Dissolving the lump and increasing the heat transfer rate are the privilege ways for controlling the artery flow[2]. The other way is destroying the vortexes which create after the lump. Because these vortexes interfere with blood flow and restrict the main blood flow[3]. The magnetic field does this target by applying an extra force to biofluid such as Synovial Fluid[4] and Spinal Fluid. During the last decades, numerous investigations have been done in finding the characteristic of blood flow in artery. In [5] the effect of magnetic field on stereotyping the blood flow in the stenosis artery is discussed. The effect of multi stenosis on the blood flow is brought up in [10, 11]. In [12] different models of macro blood flow is coupled with various area and number of stenosis. The time dependent heat transfer of two-dimensional artery model is considered in paper [13]. Our paper, introduces the effect of Lorentz and magnetization forces on the stenosis artery for uniforming biofluid flow after the lump.

2 GOVERNING EQUATIONS

Let us consider the two-dimensional flow of a non-Newtonian, steady state, viscous and incompressible flow inside a stenosis artery. The geometry of stenosis with porous walls is shown in Figure1 and assumed as below function[14]:

$$\frac{y}{R_0} = \begin{cases} 1 - \frac{b}{2R_0} \left[1 + \cos \left( \frac{\pi (x - x_0)}{x_0} \right) \right] & L_x \leq x \leq 2x_0 + L_y \\ 1 & otherwise \end{cases}$$
Where \( x_1 \) is the center of stenosis, \( x_0 \) is the half of the stenosis region width and \( h \) is the width of artery. The up and down vessel walls are modeled as porous material. This is acceptable with actual behavior of our artery blood. As depicted in Figure 1, \( h \) is the maximum height of stenosis. Increasing this parameter creates critical situation for blood flow in artery. Under the action of magnetic field the governing equations on the biofluid in the free and porous regions are described by Cauchy and Brinkman equations which are explained below.

### 2.1 Heat transfer and fluid flow

In this paper the non-dimensional variables are defined at first.

\[
x = \frac{x}{R_0}, \quad y = \frac{y}{R_0}, \quad u = \frac{u}{u_0}, \quad v = \frac{v}{v_0}, \quad p = \frac{p}{p_0^2}, \quad H = \frac{H}{H_0},
\]

\[
T = \frac{T}{T_0}
\]

Where \( \alpha = k / \rho c p \) and \( v_0 = \alpha / R_0 \) are the thermal diffusivity and characteristic velocity of the fluid. The non-dimensional Cauchy equations are expressed as below functions:

- Continuity:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

- \( x \)-momentum:

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} + \text{Pr} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + (Fx)_M + (Fx)_L
\]

- \( y \)-momentum:

\[
\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial p}{\partial y} + \text{Pr} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + (Fy)_M + (Fy)_L
\]

The Energy equation for the biofluid explained by:

\[
u \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\partial T}{\partial y} = \text{Ec} \times \frac{\partial^2 T}{\partial y^2} + \left( \frac{\partial T}{\partial x} \right)^2 - D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \left( \frac{\partial C}{\partial x} \right)^2
\]

The Brinkman equations for momentum formula inside the porous region are:

- \( x \)-momentum:

\[
0 = -\frac{\partial p}{\partial x} + \text{Pr} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + \text{Pr} \left[ \frac{\partial Da}{\partial x} \times u_p + (Fx)_M + (Fx)_L \right]
\]

- \( y \)-momentum:

\[
0 = -\frac{\partial p}{\partial y} + \text{Pr} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + \text{Pr} \left[ \frac{\partial Da}{\partial y} \times v_p + (Fy)_M + (Fy)_L \right]
\]

\( \vec{V} = (u, v) \) and \( \vec{V}_p = (u_p, v_p) \) are the two dimensional velocity field inside the free and porous regions. The third and fourth terms in the momentum and energy equations are appeared due to Magneto Hydro Dynamic (MHD) and Ferro Hydro Dynamic (FHD) impression and they are equal to:

\[
(Fx)_M = M_n \mu_m H \frac{\partial H}{\partial x}, \quad (Fx)_L = \frac{M_n}{\text{Re}} (\chi_m + 1)^2 (uH_x H_y - uH_x^2)
\]

\[
(Fy)_M = M_n \mu_m H \frac{\partial H}{\partial y}, \quad (Fy)_L = \frac{M_n}{\text{Re}} (\chi_m + 1)^2 (uH_x H_y - uH_y^2)
\]

\[
Q_M = -M_n \mu_m \times \frac{\partial \chi}{\partial T} \times H \times (u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y})
\]

\[
Q_L = \frac{M_n}{\text{Re}} \times \frac{\partial \chi}{\partial T} \times (\chi_m + 1)^2 \times (uH_x H_y - vH_y^2)^2
\]

The non-Newtonian behavior of biofluid, in this paper is modeled by power law formula \([15]\):

\[
\eta = \eta_0 \left| \frac{\dot{\gamma}}{\dot{\gamma}_n} \right|^n
\]

The value of \( \eta_0 \) and \( n \) are cited in Table 1.

### 2.2 The magnetic field intensity

The magnetization property \( (M) \) is determined the impression of magnetic field on the ferrofluid. Among numerous formula, the linear equation which depends on the magnetic field intensity and temperature is used in this paper\([16]\).

\[
M = \chi_m H
\]

\( \chi_m \) is the magnetic susceptibility that varies with temperature:
\[ \chi_m = \frac{z_0}{1 + (\beta T^*) (T - T_0) T} \]  
(13)

\[ \chi_0, \beta \text{ and } T_0 \text{ are constant parameters and electrical current through two wires is generated the magnetic field. The wires are parallel and its magnetic field intensity is express by:} \]

\[ H_x = \frac{(x - a)}{((x - a)^2 + (y - b)^2)} \]

\[ H_y = \frac{(y - b)}{((x - a)^2 + (y - b)^2)} \]  
(14)

\( H_0 \) is the magnetic field strength which depends on the applied magnetics induction \((B=\mu_0 (H+M))\) and \( a, b \) are the location of wires. The non-dimensional parameters which appear in the above equations are:

- Reynolds number:

\[ \text{Re} = \frac{R_0 \rho v_0}{\eta_0 (\frac{\alpha}{R_0})^{-0.4}} = \frac{\rho \alpha}{\eta_0 (\frac{\alpha}{R_0})^{-0.4}} \]  
(15)

- Eckert number:

\[ Ec = \frac{v_0^2}{c_p T^*} = \frac{\alpha^2}{c_p T^* R_0^2} \]  
(16)

- Prandtl number:

\[ Pr = \frac{\eta_0 (\frac{\alpha}{R_0})^{-0.4}}{\rho \alpha} \]  
(17)

- Darcy number:

\[ Da = \frac{R_0^2}{k_{br}} \]  
(18)

- Magnetic number of FHD:

\[ Mn_F = \frac{\mu_0 H_0^2}{\rho v_0^2} = \frac{\mu_0 H_0^2 R_0^2}{\rho \alpha^2} \]  
(19)

- Magnetic number of MHD:

\[ Mn_M = \frac{\mu_0^2 H_0^2 R_0^2}{\eta} \]  
(20)

2.3 Boundary conditions

The systems of equations 11-11 are elliptic and require boundary condition: On the upper and lower arterial walls:

\[ T = T_1, u = v = 0 \]  
(21)

\[ x = 7 \quad 0 < y < 1 \quad \frac{\partial T}{\partial x} = 0, p = 0 \]  
(22)

\[ x = 0 \quad 0 < y < 1 \quad T = T_1, u = 1500(y - y^2) \]  
(23)

For obtaining a continuous velocity field between the free and porous interface, the velocity is equal. All the constant parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Numerical value</th>
<th>Parameter(s)</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr\text{blood}</td>
<td>4.56</td>
<td>Ec\text{blood}</td>
<td>7.378×10^{-6}</td>
</tr>
<tr>
<td>Pr\text{wall}</td>
<td>4.04</td>
<td>Da</td>
<td>100-400-800</td>
</tr>
<tr>
<td>\beta</td>
<td>5.6×10^{-3}</td>
<td>Mn\text{M}</td>
<td>150</td>
</tr>
<tr>
<td>Re\text{wall}</td>
<td>0.247</td>
<td>Mn\text{F}</td>
<td>[105:5×10^{-5}]</td>
</tr>
<tr>
<td>Re\text{blood}</td>
<td>0.23</td>
<td>Zo</td>
<td>0.06</td>
</tr>
<tr>
<td>T_1</td>
<td>300 0 K</td>
<td>b_1</td>
<td>-0.1</td>
</tr>
<tr>
<td>n</td>
<td>0.6</td>
<td>a_1</td>
<td>3.5</td>
</tr>
<tr>
<td>\eta_0</td>
<td>35×10^{-3}</td>
<td>b_2</td>
<td>1.1</td>
</tr>
<tr>
<td>Ec\text{wall}</td>
<td>1.05×10^{-5}</td>
<td>a_2</td>
<td>3.5</td>
</tr>
</tbody>
</table>

The velocity and temperature contours in the stenosis region are depicted in Figure 2. Around this region the velocity becomes significantly high corresponds to lower cross sectional area. As the streamlines show after the stenosis region two vortexes are created near the up and down walls. These vortexes can disrupt the blood flow. The temperature contour shows that the temperature is high in the middle of vessel due to linear dependency of temperature to velocity. As the velocity is maximized in stenosis region the temperature becomes high in this region too. The figure shows the temperature is high in the communal interface of porous and free region. This is result of higher Eckert number of porous walls versus free space.
Figure 2. The velocity and temperature contours of stenosis artery without magnetic effect.

3 SIMULATION RESULTS
The velocity streamline and temperature contours in present of three different value of magnetic field intensity, are demonstrated in Figure 3. The primary effect of exerting magnetic field to the biomagnetic fluid is destroying the formed vortex approximately after the stenosis region.
This magnetic field creates two vortices in opposite direct, and neutralize the formed vortices after the lump. With increasing the magnetic field intensity, formed vortices by magnetic field expand, till they overcome the flow vortices and two vortices in opposite direction are created. So, the magnetic intensity is important and should be optimum for reaching the target goal. In this paper, the best option for magnetic intensity is $10^{12}$. Figure 3 shows by increasing magnetic field intensity, the flow vortices are destroyed almost which can result better and smooth blood flow in vessel. Temperature contours for different magnetic field intensity are also depicted in figure. This shows that the powerful magnetic field can focus the temperature near the wire place due to heat effect of wire in blood. The temperature with ten times higher magnetic field intensity from $10^{10}$ and $10^{11}$ becomes 1.2X and 3.5X respectively.

Shear stress on the low wall is demonstrated in Figure 4. By strengthening the magnetic field, the bigger vortices which can influence wider area are created, therefore main vortex is destroyed completely and the velocity gradient decreases and lower shear stress value is applied to the wall. By applying magnetic field with ten times stronger than $10^{11}$, the shear stress will become 6.6X lower in vortex created region. The magnetic forces significantly affect the biofluid pressure. Increasing the magnetic field intensity causes the pressure after the lump declines due to neutralizing the vortex and calm current creation. This fact is shown in Figure 5 that by increasing the magnetic field from $10^{11}$ to $10^{12}$, the maximum pressure becomes 8.6X lower. Maximum temperature and pressure versus changing in porosity of walls and the magnetic field are archived in Table 2. The Darcy number indicates the porosity of walls which higher value of it results lower porous characteristic of walls and nearing it to rigid walls. As mentioned before, the magnetic field acts as a heat source around the wire current. This field has more effect on porous media than solid walls due to lower heat transfer of...
high porosity media. The table shows that increasing the magnetic field intensity produces higher pressure and temperature in artery such that with applying \(10^{12}\) the magnetic field (Mnf), the temperature and pressure increase 2.7X and 4.1X respect with neglecting magnetic effect.

### 4 CONCLUSION

This paper introduce the impact of magnetic field on a Non-Newtonian biomagnetic fluid in stenosis artery. The magnetic field is used for destroying the created vortex after the stenosis region corresponds to non-uniforming of flow in this region. As results shows the magnetic field can neutralize this vortexes by producing opposite vortexes in this region. Furthermore the magnetic field decrease the pressure and shear stress in this region due to creating the calm flow. The results shows that changing in temperature and pressure reduction is significant related to the value of magnetic field intensity.

**Table 3. Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>Pressure</td>
</tr>
<tr>
<td>(T)</td>
<td>Temperature</td>
</tr>
<tr>
<td>(T_a)</td>
<td>Arterial temperature</td>
</tr>
<tr>
<td>(R_0)</td>
<td>Vessel diameter</td>
</tr>
<tr>
<td>(h)</td>
<td>Height of stenosis</td>
</tr>
<tr>
<td>(L_s)</td>
<td>Half-length of stenosis</td>
</tr>
<tr>
<td>(L_T)</td>
<td>Length of artery</td>
</tr>
<tr>
<td>(H)</td>
<td>Magnetic field strength</td>
</tr>
<tr>
<td>(B)</td>
<td>Magnetic field induction</td>
</tr>
<tr>
<td>(M)</td>
<td>Magnetization of fluid</td>
</tr>
<tr>
<td>(J)</td>
<td>Density of electrical current</td>
</tr>
<tr>
<td>(M_{nf})</td>
<td>Magnetic number (FHD)</td>
</tr>
<tr>
<td>(M_{m})</td>
<td>Magnetic number (MHD)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Porosity of porous media</td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>Magnetic permeability of vacuum</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Dynamic viscosity</td>
</tr>
</tbody>
</table>

**Table 2. Maximum temperature and pressure in different porosity and \(M_{nf}=10^2\)**

<table>
<thead>
<tr>
<th>Da</th>
<th>40</th>
<th>400</th>
<th>4000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{nf})</td>
<td>(P\times 10^6)</td>
<td>(T)</td>
<td>(P\times 10^6)</td>
</tr>
<tr>
<td>0</td>
<td>10.47</td>
<td>0.470</td>
<td>10.55</td>
</tr>
<tr>
<td>(10^{10})</td>
<td>10.80</td>
<td>0.761</td>
<td>10.73</td>
</tr>
<tr>
<td>(10^{11})</td>
<td>12.59</td>
<td>0.997</td>
<td>12.40</td>
</tr>
<tr>
<td>(10^{12})</td>
<td>31.71</td>
<td>4.37</td>
<td>29.11</td>
</tr>
</tbody>
</table>

**Figure 4.** Shear stress in the vortex place with different magnetic field intensity and \(M_{nf}=10^2\).

**Figure 5.** Pressure distribution in stenosis artery with different magnetic field intensity and \(M_{nf}=10^2\).

I. REFERENCES


